

Year 7 Term 2

Factors, Multiples and Primes

What do I need to be able to do?

By the end of this unit you should be able to:

- Find and use multiples
- Identify factors of numbers and expressions
- Recognise and identify prime numbers
- Recognise square and triangular numbers
- Find common factors including HCF
- Find common multiples including LCM

Keywords

Multiples: found by multiplying any number by positive integers

Factor: integers that multiply together to get another number.

Prime: an integer with only 2 factors

Conjecture: a statement that might be true (based on reasoning) but is not proven

Counterexample: a special type of example that disproves a statement

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

HCF: highest common factor (biggest factor two or more numbers share)

LCM: lowest common multiple (the first time the times table of two or more numbers match)

Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3

3, 6, 9, 12, 15...

$3x, 6x, 9x \dots$

This list continues and doesn't end

Non example of a multiple

45 is not a multiple of 3 because it is 3×15

Not an integer

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Factors

Arrays can help represent factors

5×2 or 2×5

Factors of 10
1, 2, 5, 10

10×1 or 1×10

Factors and expressions

$x \ x \ x \ x \ x \ x$

The number itself is always a factor

Factors of $6x$

$6, x, 1, 6x, 2x, 3, 3x, 2$

$6x \times 1$ OR $6 \times x$

$x \ x$
 $x \ x$

$2x \times 3$

$x \ x \ x$
 $x \ x \ x$

$3x \times 2$

Prime numbers

- Integer
- Only has 2 factors
- and itself

The first prime number
The only even prime number

2

Learn or how-to quick recall...

2, 3, 5, 7, 11, 13, 17, 19, 23, 29...

Square and triangular numbers

Square numbers

odd even odd

Representations are useful to understand a square number n^2

1, 4, 9, 16, 25, 36, 49, 64 ...

Triangular numbers

Representations are useful – an extra counter is added to each new row

Add two consecutive triangular numbers and get a square number

1, 3, 6, 10, 15, 21, 28, 36, 45...

Common factors and HCF

Common factors are factors two or more numbers share

HCF – Highest common factor

HCF of 18 and 30

18 1, 2, 3, 6, 9, 18

30 1, 2, 3, 5, 6, 10, 15, 30

Common factors
(factors of both numbers)
1, 2, 3, 6

HCF = 6

6 is the biggest factor they share

Common multiples and LCM

Common multiples are multiples two or more numbers share

LCM – Lowest common multiple

LCM of 9 and 12

LCM = 36

The first time their multiples match

9 9, 18, 27, 36, 45, 54

12 12, 24, 36, 48, 60

9 12 27 36 45

Comparing fractions

$\frac{3}{5}$ and $\frac{7}{10}$

Compare fractions using a LCM denominator

$\frac{6}{10}$ and $\frac{7}{10}$

Conjectures and counterexamples

Conjecture

1, 2, 4, ...

The numbers in the sequence are doubling each time.

Counterexamples



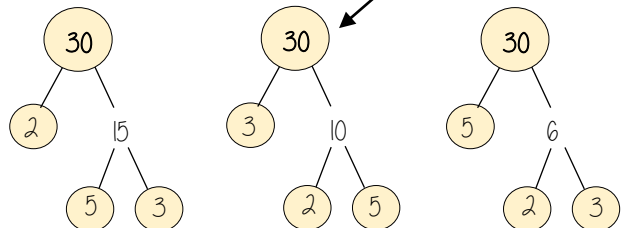
This sequence isn't doubling it is adding 2 each time

A pattern that is noticed for many cases

Only one counterexample is needed to disprove a conjecture

Product of prime factors

Multiplication part-whole models



All three prime factor trees represent the same decomposition

Multiplication is commutative

$30 = 2 \times 3 \times 5$

Multiplication of prime factors

Using prime factors for predictions

e.g 60 30×2 $2 \times 3 \times 5 \times 2$
150 30×5 $2 \times 3 \times 5 \times 5$

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Fractions

What do I need to be able to do?

By the end of this unit you should be able to:

- Convert between mixed numbers and fractions
- Add/Subtract unit fractions (same denominator)
- Add/Subtract fractions (same denominator)
- Add/Subtract fractions from integers
- Use equivalent fractions
- Add/Subtract any fractions
- Add/Subtract improper fractions and mixed numbers
- Use fractions in algebraic contexts

Keywords

Numerator: the number above the line on a fraction. The top number. Represents how many parts are taken

Denominator: the number below the line on a fraction. The number represents the total number of parts

Equivalent: of equal value

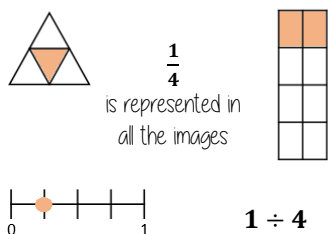
Mixed numbers: a number with an integer and a proper fraction

Improper fractions: a fraction with a bigger numerator than denominator

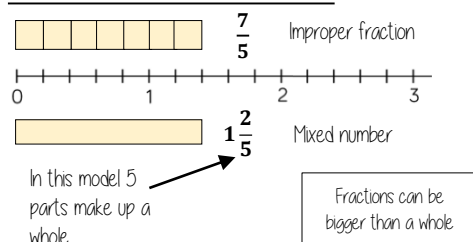
Substitute: replace a variable with a numerical value

Place value: the value of a digit depending on its place in a number. In our decimal number system, each place is 10 times bigger than the place to its right

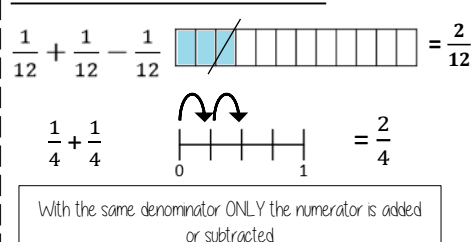
Representing Fractions



Mixed numbers and fractions

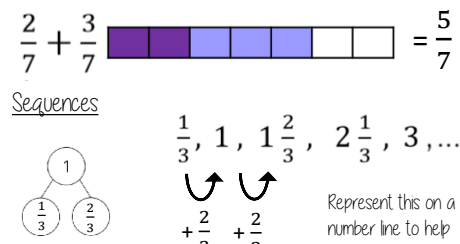


Add/Subtract unit fractions

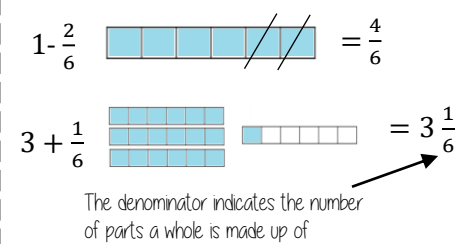


Add/Subtract fractions

Same denominator

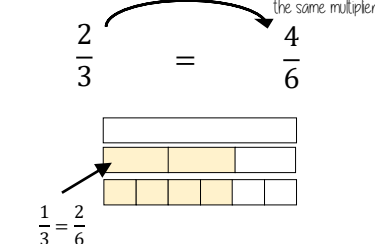


Add/Subtract from integers

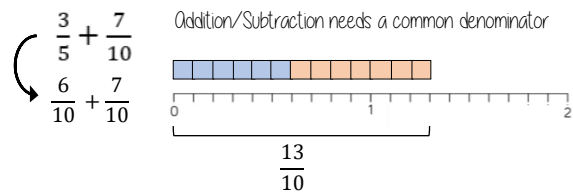


Equivalent fractions

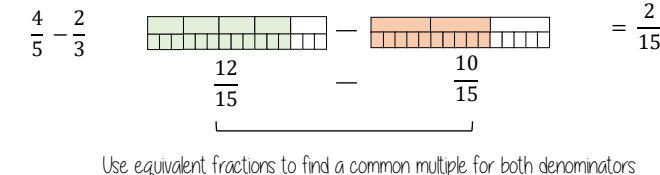
Numerator and denominator have the same multiplier



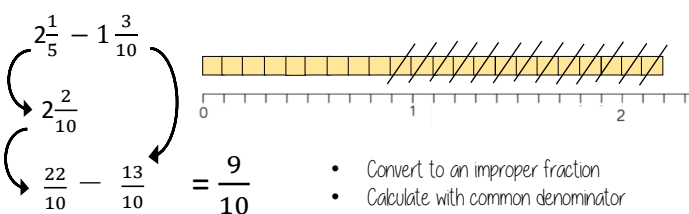
Add/Subtraction fractions (common multiples)



Add/Subtraction any fractions



Add/Subtraction fractions (improper and mixed)

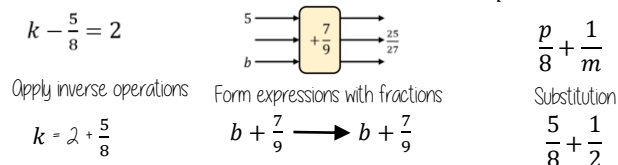


Partitioning method

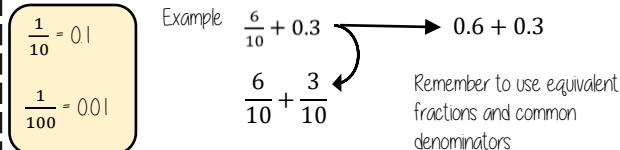
$$2\frac{1}{5} - 1\frac{3}{10} = 2\frac{2}{10} - 1\frac{3}{10} = 2\frac{2}{10} - 1 - \frac{3}{10} = 1\frac{2}{10} - \frac{3}{10} = \frac{9}{10}$$

Fractions in algebraic contexts

$p = 5 \quad m = 2$



Fractions and decimals



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Brackets

What do I need to be able to do?

By the end of this unit you should be able to:

- Form Expressions
- Expand and factorise single brackets
- Form and solve equations
- Solve equations with brackets
- Represent inequalities
- Form and solve inequalities

Keywords

Simplify: grouping and combining similar terms

Substitute: replace a variable with a numerical value

Equivalent: something of equal value

Coefficient: a number used to multiply a variable

Product: multiply terms

Highest Common Factor (HCF): the biggest factor (or number that multiplies to give a term)

Inequality: an inequality compares two values showing if one is greater than, less than or equal to another

Form expressions

For unknown variables, a letter is normally used in its place

More than – ADD

Less than/ difference – SUBTRACT

e.g. 4 more than $t \longrightarrow t + 4$

8 less than $k \longrightarrow k - 8$

Only similar terms can be grouped together

e.g. Find the perimeter of this shape
(Perimeter = length around outside of shape)

$$t + 2t + 1 + t + 2t + 1 \longrightarrow 6t + 2$$

Directed numbers

$$+ + \longrightarrow +$$

$$- - \longrightarrow +$$

$$+ - \longrightarrow -$$

$$- + \longrightarrow -$$

e.g. $a = -5$ and $b = 2$

$$a^2 = a \times a = -5 \times -5 = 25$$

$$b + a = 2 + -5 = -3$$

Multiply single brackets

$$3(2x + 4)$$

$$\begin{array}{|c|c|} \hline 2x & 4 \\ \hline 3 \times 2x & 3 \times 4 \\ \hline 6x & 12 \\ \hline \end{array}$$

$$6x + 12$$

$$6x + 12$$

$$\begin{array}{|c|c|c|} \hline 2x + 4 & 2x + 4 & 2x + 4 \\ \hline x & x & x \\ \hline 4 & 4 & 4 \\ \hline \end{array}$$

$$6x + 12$$

Different representations of $3(2x + 4) = 6x + 12$

Factorise into a single bracket

$$8x + 4$$

$$\begin{array}{|c|} \hline 8x + 4 \\ \hline \end{array}$$

$$2x + 1$$

Try and make this the highest common factor

The two values multiply together (also the area) of the rectangle

$$8x + 4 \equiv 4(2x + 1)$$

Note:

$$8x + 4 \equiv 2(4x + 2)$$

This is factorised but the HCF has not been used

Solve equations with brackets

$$3(2x + 4) = 30$$

$$\begin{array}{|c|c|c|} \hline 2x + 4 & 2x + 4 & 2x + 4 \\ \hline \end{array}$$

$$30$$

$$\begin{array}{|c|c|c|} \hline x & x & x \\ \hline 4 & 4 & 4 \\ \hline \end{array}$$

$$30$$

$$\begin{array}{|c|c|c|} \hline x & x & x \\ \hline \end{array}$$

$$12$$

$$30$$

$$\begin{array}{|c|c|c|} \hline x & x & x \\ \hline \end{array}$$

$$18$$

$$3(2x + 4) = 30$$

Expand the brackets

$$6x + 12 = 30$$

$$-12$$

$$-12$$

$$6x = 18$$

$$-6$$

$$-6$$

Substitute to check your answer.
This could be negative or a fraction or decimal

$$\begin{array}{|c|} \hline x \\ \hline 3 \\ \hline \end{array} x = 3$$

Simple Inequalities

< less than

≤ Less than or equal to

> More than

≥ More than or equal to

$$x < 10$$

Say this out loud
"x is a value less than 10"

Note:
 $x < 10$ and $10 > x$
represent the same values

$$x + 2 \leq 20$$

"my value + 2 is less than or equal to 20"

$$x \leq 18$$

The biggest the value can be is 18

$$10 > x$$

Say this out loud
"10 is more than the value"

Form and solve inequalities



Two more than treble my number is greater than 11

Find the possible range of values

Form

$$x \longrightarrow x \times 3 \longrightarrow +2 \longrightarrow 11$$

$$3x + 2 > 11$$

Solve

$$x \longleftarrow -3 \longleftarrow -2 \longleftarrow 11$$

$$x > 3$$

Check

This would suggest any value bigger than 3 satisfies the statement

$$3 \times 3 + 2 = 11 \checkmark$$

$$10 \times 3 + 2 = 32 \checkmark$$

Algebraic constructs

Expression

A sentence with a minimum of two numbers and one maths operation

Equation

A statement that two things are equal

Term

A single number or variable

Identity

An equation where both sides have variables that cause the same answer includes $\equiv$

Formula

A rule written with all mathematical symbols
e.g. area of a rectangle $A = b \times h$