

Year 11 Higher Term 1

More Algebra

What do I need to be able to do?

By the end of this unit you should be able to:

- Draw quadratic graphs
- Interpret quadratic graphs
- Interpret other graphs including reciprocals
- Represent inequalities

Keywords

Quadratic: a curved graph with the highest power being 2. Square power.

Inequality: makes a non equal comparison between two numbers

Reciprocal: a reciprocal is 1 divided by the number

Cubic: a curved graph with the highest power being 3. Cubic power.

Origin: the coordinate (0, 0)

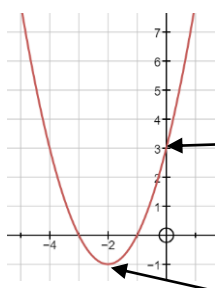
Parabola: a 'u' shaped curve that has mirror symmetry

Quadratic Graphs

$$y = x^2 + 4x + 3$$

If x^2 is the highest power in your equation then you have a quadratic graph

It will have a parabola shape



Substitute the x values into the equation of your line to find the y coordinates

x	-4	-3	-2	-1	0	1
y	3	0	-1	0	3	8

Coordinate pairs for plotting $(-3, 0)$

Plot all of the coordinate pairs and join the points with a curve (freehand)

Quadratic graphs are always symmetrical with the turning point in the middle

Interpret other graphs

Cubic Graphs

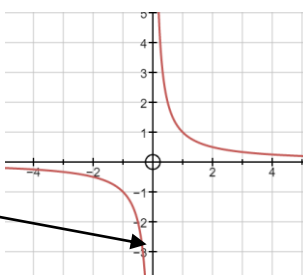
$$y = x^3 + 2x^2 - 2x + 1$$



If x^3 is the highest power in your equation then you have a cubic graph

Reciprocal Graphs

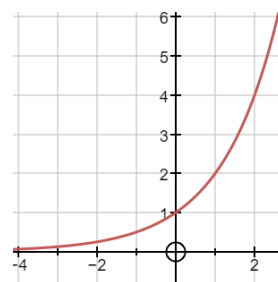
$$y = \frac{1}{x}$$



Reciprocal graphs never touch the y axis
This is because x cannot be 0
This is an asymptote

Exponential Graphs

$$y = 2^x$$



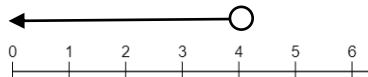
Exponential graphs have a power of x

Represent Inequalities

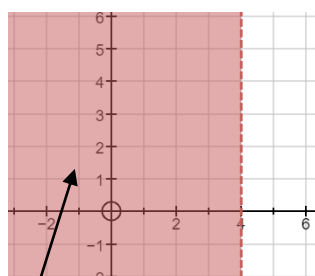
Multiple methods of representing inequalities

$$x < 4$$

All values are less than 4



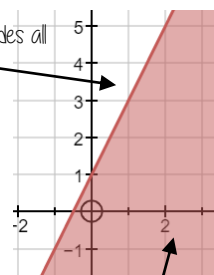
The shaded area indicates all possible values of x



The dotted line shows that the inequality does not include these points

The solid line shows that the inequality includes all the points on this line

$$y \geq 2x + 1$$



The shaded area indicates all possible solutions to this inequality

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More Algebra

What do I need to be able to do?

By the end of this unit you should be able to:

- Determine whether (x,y) is a solution
- Solve by substituting a known variable
- Solve by substituting an expression
- Solve graphically
- Solve by subtracting/ adding equations
- Solve by adjusting equations
- Form and solve linear simultaneous equations

Keywords

Solution: a value we can put in place of a variable that makes the equation true

Variable: a symbol for a number we don't know yet

Equation: an equation says that two things are equal – it will have an equals sign =

Substitute: replace a variable with a numerical value

LCM: lowest common multiple (the first time the times table of two or more numbers match)

Eliminate: to remove

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

Coordinate: a set of values that show an exact position

Intersection: the point two lines cross or meet

Is (x, y) a solution?

x and y represent values that can be substituted into an equation

Does the coordinate (1,8) lie on the line $y=3x+5$?

This coordinate represents $x=1$ and $y=8$

$$y = 3x + 5$$

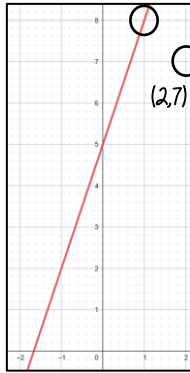
$$8 = 3(1) + 5$$

As the substitution makes the equation correct the coordinate (1,8) IS on the line $y=3x+5$

Is (2,7) on the same line?

$$7 \neq 3(2) + 5$$

No 7 does NOT equal $6+5$



Substituting known variables

A line has the equation $3x + y = 14$

Two different variables, two solutions

Stephanie knows the point $x = 4$ lies on that line. Find the value for y

$$x = 4$$

$$3x + y = 14$$

$$3(4) + y = 14$$

$$12 + y = 14$$

$$-12 \quad -12$$

$$y = 2$$

Substituting in an expression

$$x = 2y$$

$$x + y = 30$$

Pair of simultaneous equations (two representations)

Substitute $2y$ in place of the x variable as they represent the same value

$$x = 2y$$

$$x + y = 30$$

$$3y = 30$$

$$y = 10$$

$$3y = 30$$

$$\div 3 \quad \div 3$$

$$y = 10$$

$$x = 2y$$

$$x = 20$$

Solve graphically

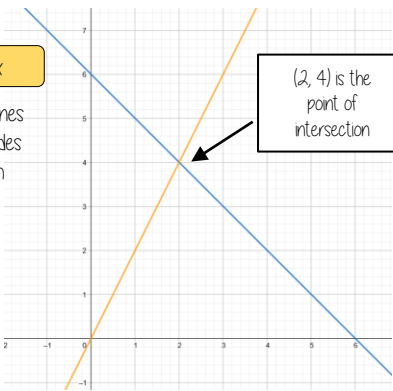
$$x + y = 6$$

$$y = 2x$$

Linear equations are straight lines
The point of intersection provides the x and y solution for both equations

The solution that satisfies both equations is

$$x = 2 \text{ and } y = 4$$



Solve by subtraction

$$18$$

$$10$$

$$8$$

$$x = 4$$

$$y = 3$$

$$3x + 2y = 18$$

$$x + 2y = 10$$

$$2x = 8$$

$$\div 2 \quad \div 2$$

$$x = 4$$

$$x + 2y = 10$$

$$(4) + 2y = 10$$

$$-4 \quad -4$$

$$2y = 6$$

$$\div 2 \quad \div 2$$

$$y = 3$$

$$x + x + x + y + y = 18$$

$$x + y + y = 10$$

$$x + x + y + y = 18$$

$$x + y + y = 10$$

$$x + x = 8$$

$$x = 4$$

$$y = 3$$

Solve by addition

Addition makes zero pairs

$$3x + 2y = 16$$

$$+ 6x - 2y = 2$$

$$9x = 18$$

$$\div 9 \quad \div 9$$

$$x = 2$$

$$3x + 2y = 16$$

$$3(2) + 2(y) = 16$$

$$6 + 2y = 16$$

$$-6 \quad -6$$

$$2y = 10$$

$$y = 5$$

$$x = 2$$

$$y = 5$$

Solve by adjusting one

$$h + j = 12$$

$$2h + 2j = 29$$

$$2h + 2j = 24$$

$$2h + 2j = 29$$

By proportionally adjusting one of the equations – now solve the simultaneous equations choosing an addition or subtraction method

$$12$$

$$24$$

$$29$$

Solve by adjusting both

$$2x + 3y = 39$$

$$5x - 2y = -7$$

Use LCM to make equivalent x OR y values
Because of the negative values using zero pairs and y values is chosen choice

$$4x + 6y = 78$$

$$15x - 6y = -21$$

Now solve by addition

$$x = 2$$

$$y = 5$$

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More Algebra

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve inequalities with negative numbers
- Solve equations with unknowns on both sides
- Solve inequalities with unknowns on both sides
- Substitute into formulae and equations
- Rearrange formulae

Keywords

Inequality: an inequality compares two values showing if one is greater than, less than or equal to another

Variable: a quantity that may change within the context of the problem

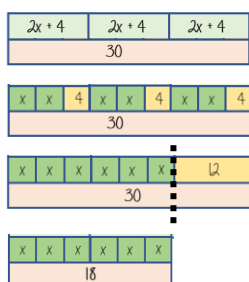
Rearrange: Change the order

Inverse operation: the operation that reverses the action

Substitute: replace a variable with a numerical value

Solve: find a numerical value that satisfies an equation

Solve equations with brackets



$$3(2x + 4) = 30$$

Expand the brackets

$$6x + 12 = 30$$

$$-12 \quad -12$$

$$6x = 18$$

$$-6 \quad -6$$

$$x = 3$$

Form and solve inequalities



Two more than treble my number is greater than 11

Find the possible range of values

$$3x + 2 > 11$$

Solve

$$x \leftarrow -3 \leftarrow -2 \leftarrow 11$$

$$x > 3$$

Inequalities with negatives

Method 1 Make x positive first

$$2 - 3x > 17$$

$$+3x \quad +3x$$

$$2 > 17 + 3x$$

$$-17 \quad -17$$

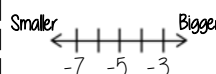
$$-15 > 3x$$

$$\div 3 \quad \div 3$$

$$-5 > x$$

x is true for any value smaller than -5

CHECK IT!
 $2 - 3(-6) = 20$
TRUE/ CORRECT



Equations with unknown on both sides

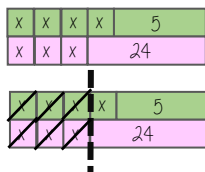
$$4x + 5 = 3x + 24$$

$$-3x \quad -3x$$

$$x + 5 = 24$$

$$-5 \quad -5$$

$$x = 19$$



Inequalities with unknown on both sides

Solving inequalities has the same method as equations

$$5(x + 4) < 3(x + 2)$$

$$5x + 20 < 3x + 6$$

$$2x + 20 < 6$$

$$2x < -14$$

$$x < -7$$

Check it!

$$5(-8 + 4) < 3(-8 + 2)$$

$$5(-4) < 3(-6)$$

$$-20 < -18$$

✓ -20 IS smaller than -18

Method 2 Keep the negative x

$$2 - 3x > 17$$

$$-2 \quad -2$$

$$-3x > 15$$

$$\div -3 \quad \div -3$$

$$x > -5$$

x is true for any value bigger than -5

This cannot be true...

$$x < -5$$

When you multiply or divide x by a negative you need to reverse the inequality

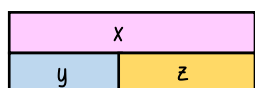
Formulae and Equations

Formulae – all expressed in symbols

Substitute in values

Equations – include numbers and can be solved

Rearranging Formulae (one step)



$$x = y + z$$

Rearrange to make y the subject

$$y = x - z$$

$$y \rightarrow +z \rightarrow x$$

$$y \leftarrow -z \leftarrow x$$

Using inverse operations or fact families will guide you through rearranging formulae

Rearranging can also be checked by substitution

Language of rearranging...

Make XXX the subject

Change the subject

Rearrange

Rearranging Formulae (two step)

In an equation (find x)

$$4x - 3 = 9$$

$$+3 \quad +3$$

$$4x = 12$$

$$\div 4 \quad \div 4$$

$$x = 3$$

In a formula (make x the subject)

$$xy - s = a$$

$$+s \quad +s$$

$$xy = a + s$$

$$\div y \quad \div y$$

$$x = \frac{a + s}{y}$$

The steps are the same for solving and rearranging

Rearranging is often needed when using $y = mx + c$

e.g Find the gradient of the line $2y - 4x = 9$

Make y the subject first $y = \frac{4x + 9}{2}$

Gradient = $\frac{4}{2} = 2$

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Vectors

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand and represent vectors
- Use and read vector notation
- Draw and understand vectors multiplied by a scalar
- Draw and understand addition of vectors
- Draw and understand addition and subtraction of vectors

Keywords

Direction: the line our course something is going

Magnitude: the magnitude of a vector is its length

Scalar: a single number used to represent the multiplier when working with vectors

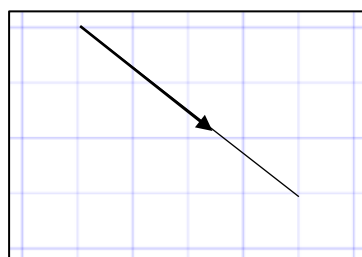
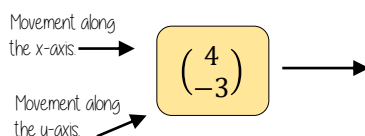
Column vector: a matrix of one column describing the movement from a point

Resultant: the vector that is the sum of two or more other vectors

Parallel: straight lines that never meet

Understand and represent vectors

Column vectors have been seen in translations to describe the movement of one image onto another



Vectors show both direction and magnitude

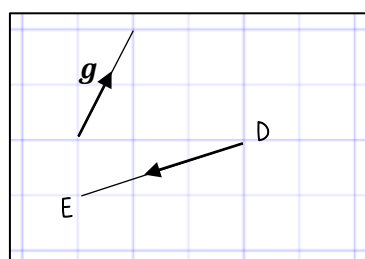
The arrow is pointing in the direction from starting point to end point of the vector.

The direction is important to correctly write the vector

The magnitude is the length of the vector (This is calculated using Pythagoras theorem and forming a right-angled triangle with auxiliary lines)

The magnitude stays the same even if the direction changes

Understand and represent vectors



Vector notation $\overrightarrow{DE}$ is another way to represent the vector joining the point D to the point E

$$\overrightarrow{DE} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$$

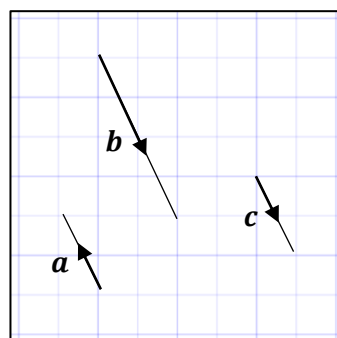
The arrow also indicates the direction from point D to point E

Vectors can also be written in bold lower case so $\mathbf{g}$ represents the vector

$$\mathbf{g} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Vectors multiplied by a scalar

Parallel vectors are scalar multiples of each other



$$\mathbf{b} = 2 \times \mathbf{c} = 2\mathbf{c}$$

Multiply $\mathbf{c}$ by 2 this becomes $\mathbf{b}$.
The two lines are parallel

$$\mathbf{a} = -1 \times \mathbf{c} = -\mathbf{c}$$

The vectors $\mathbf{a}$ and $\mathbf{c}$ are also parallel. A negative scalar causes the vector to reverse direction

$$\mathbf{b} = -2 \times \mathbf{a} = -2\mathbf{a}$$

$$\mathbf{a} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

Addition of vectors

$$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\overrightarrow{BC} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

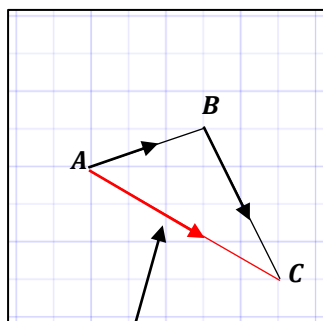
$$\overrightarrow{AB} + \overrightarrow{BC}$$

$$= \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} 3+2 \\ 1+(-4) \end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

Look how this addition compares to the vector $\overrightarrow{AC}$



The resultant

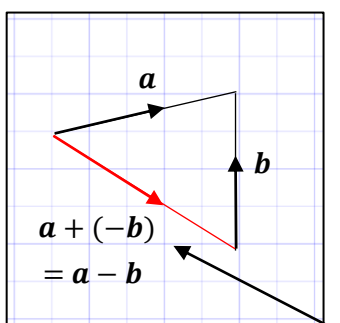
$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

Addition and subtraction of vectors

$$\mathbf{a} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$$\mathbf{b} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

$$\mathbf{a} + (-\mathbf{b}) = \begin{pmatrix} 5 & -0 \\ 1 & -4 \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$$



$$\mathbf{a} + (-\mathbf{b}) = \mathbf{a} - \mathbf{b}$$

The resultant is $\mathbf{a} - \mathbf{b}$ because the vector is in the opposite direction to $\mathbf{b}$ which needs a scalar of -1