

Year 9 Term 2

3D Shapes

What do I need to be able to do?

By the end of this unit you should be able to:

- Name 2D & 3D shapes
- Recognise Prisms
- Sketch and recognise nets
- Draw plans and elevations
- Find areas of 2D shapes
- Find Surface area for cubes, cuboids, triangular prisms and cylinders
- Find the volume of 3D shapes

Keywords

2D: two dimensions to the shape e.g length and width

3D: three dimensions to the shape e.g length, width and height

Vertex: a point where two or more line segments meet

Edge: a line on the boundary joining two vertex

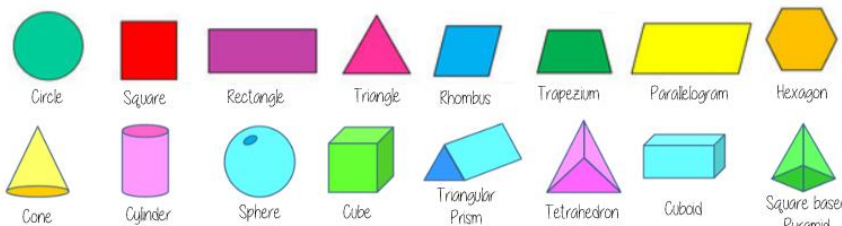
Face: a flat surface on a solid object

Cross-section: a view inside a solid shape made by cutting through it

Plan: a drawing of something when drawn from above (sometimes birds eye view)

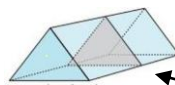
Perspective: a way to give illustration of a 3D shape when drawn on a flat surface.

Name 2D & 3D shapes



Recognise prisms

A solid object with two identical ends and flat sides

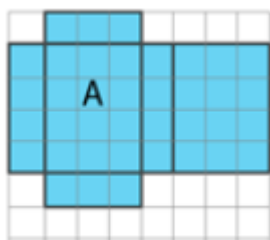
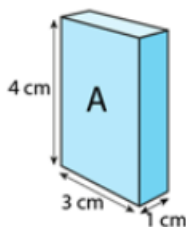


The cross section will also be identical to the end faces



A cylinder although with very similar properties does not have flat faces so is not categorised as a prism

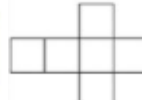
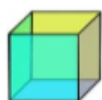
Nets of cuboids



1cm grids help to draw accurately

Visualise the folding of the net. Will it make the cuboid with all sides touching

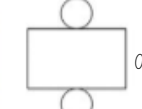
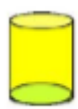
Sketch and recognise nets



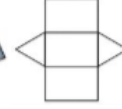
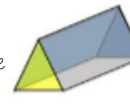
Do they have the same number of faces?



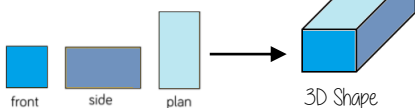
Where do the edges join?



Are the shapes of the faces correct?



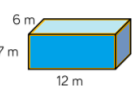
Plans and elevations



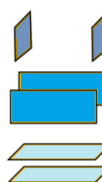
The direction you are considering the shape from determines the front and side views

Surface area

Sketching nets first helps you visualise all the sides that will form the overall surface area



For cubes and cuboids you can also find one of each face and double it



Sides 6×7
 6×7
 Front and back 12×7
 12×7
 Top and Bottom 12×6
 12×6

Sum of all sides is surface area



For other shapes - not all the sides are the same, so calculate the individually

Volumes

Volume is the 3D space it takes up - also known as capacity if using liquids to fill the space



Counting cubes

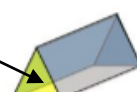
Some 3D shape volumes can be calculated by counting the number of cubes that fit inside the shape

Cubes/ Cuboids = base x width x height

Remember multiplication is commutative



Cross section



Prisms and cylinders = area cross section x height

Height can also be described as depth

Area of 2D shapes

Rectangle $\text{Base} \times \text{Height}$



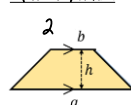
Triangle $\frac{1}{2} \times \text{Base} \times \text{Perpendicular height}$



Parallelogram/ Rhombus $\text{Base} \times \text{Perpendicular height}$



Area of a trapezium $\frac{(a+b) \times h}{2}$



Area of a circle $\pi \times \text{radius}^2$



Surface area - cylinders

The area of the circle $\pi \times \text{radius}^2$



The width of this face is the same as the circumference $\pi \times \text{diameter} \times \text{height}$

$2 \times \pi \times \text{radius}^2 + \pi \times \text{diameter} \times \text{height}$

Areas - square units
 Volumes - cube units

Areas and volumes can be left in terms of π

Year 9 Term 2

Pythagoras' theorem

What do I need to be able to do?

By the end of this unit you should be able to:

- Use square and cube roots
- Identify the hypotenuse
- Calculate the hypotenuse
- Find a missing side in a Right angled triangle
- Use Pythagoras' theorem on axes
- Explore proofs of Pythagoras' theorem

Keywords

Square number: the output of a number multiplied by itself

Square root: a value that can be multiplied by itself to give a square number

Hypotenuse: the largest side on a right angled triangle. Always opposite the right angle.

Opposite: the side opposite the angle of interest

Adjacent: the side next to the angle of interest

Squares and square roots

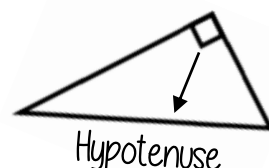
				This can also be written as 6^2					
1 x 1	2 x 2	3 x 3	4 x 4	5 x 5	6 x 6	7 x 7	8 x 8	9 x 9	10 x 10
1	4	9	16	25	36	49	64	81	100

Square numbers

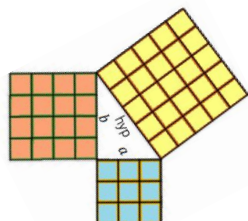
$\sqrt{\quad}$ is the square root symbol

eg $\sqrt{64} = 8$
Because $8 \times 8 = 64$

Identify the hypotenuse



Determine if a triangle is right-angled



If a triangle is right-angled, the sum of the squares of the shorter sides will equal the square of the hypotenuse.

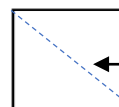
$$a^2 + b^2 = \text{hypotenuse}^2$$

eg $a^2 + b^2 = \text{hypotenuse}^2$

$$\begin{aligned} 3^2 + 4^2 &= 5^2 \\ 9 + 16 &= 25 \end{aligned}$$

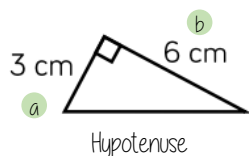
Substituting the numbers into the theorem shows that this is a right-angled triangle

The hypotenuse is always the longest side on a triangle because it is opposite the biggest angle.



Polygons can still have a hypotenuse if it is split up into triangles and opposite a right angle

Calculate the hypotenuse



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

1 Substitute in the values for a and b

$$3^2 + 6^2 = \text{hypotenuse}^2$$

$$9 + 36 = \text{hypotenuse}^2$$

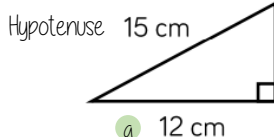
$$45 = \text{hypotenuse}^2$$

$$\sqrt{45} = \text{hypotenuse}$$

$$6.71\text{cm} = \text{hypotenuse}$$

2 To find the hypotenuse square root the sum of the squares of the shorter sides

Calculate missing sides



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

$$12^2 + b^2 = 15^2$$

1 Substitute in the values you are given

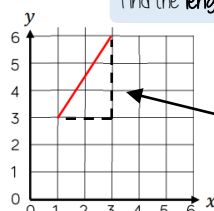
$$144 + b^2 = 225$$

Rearrange the equation by subtracting the shorter square from the hypotenuse squared

$$\begin{aligned} \text{Square root to find the length of the side} \quad & \left\{ \begin{aligned} b^2 &= 111 \\ b &= \sqrt{111} = 10.54 \text{ cm} \end{aligned} \right. \end{aligned}$$

Pythagoras' theorem on a coordinate axis

Find the length of the line segment



The segment can be made into a right-angled triangle by adding the sides on the diagram

The line segment is the hypotenuse

$$a^2 + b^2 = \text{hypotenuse}^2$$

The lengths of a and b are the sides of the triangle

Be careful to check the scale on the axes

Year 9 Term 2

ratio & proportion

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems with direct proportion
- Use conversion graphs
- Solve problems with inverse proportion
- Solve ratio problems
- Solve 'best buy' problems

Keywords

Proportion: a comparison between two numbers

Ratio: a ratio shows the relative size of two variables

Direct proportion: as one variable is multiplied by a scale factor the other variable is multiplied by the same scale factor.

Inverse proportion: as one variable is multiplied by a scale factor the other is divided by the same scale factor.

Direct Proportion

As one variable changes the other changes at the same rate.



4 cans of pop = £2.40

$\times 0.5$ 4 cans of pop = £2.40
 $\times 50$ 2 cans of pop = £1.20

This multiplier is the same in the same way that this would be for ratio

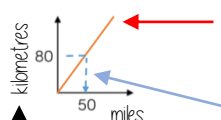
This is a multiplicative change

$\times 3$ 4 cans of pop = £2.40
 $\times 3$ 12 cans of pop = £7.20

Sometimes this is easiest if you work out how much one unit is worth first
 e.g. 1 can of pop = £0.60

Conversion Graphs

Compare two variables



Labelling of both axes is vital

This is always a straight line because as one variable increases so does the other at the same rate

To make conversions between units you need to find the point to compare – then find the associated point by using your graph
 Using a ruler helps for accuracy
 Showing your conversion lines help as a "check" for solutions

Inverse Proportion

As one variable is multiplied by a scale factor the other is divided by the same scale factor

Examples of inversely proportional relationships

Time taken to fill a pool and the number of taps running

Time taken to paint a room and the number of workers

T is inversely proportional to G. When T=2 then G=20

T	1	2	8
G	40	20	5

Arrows indicate: 1 to 2 is $\times 2$, 2 to 8 is $\times 4$, 40 to 20 is $\div 2$, 20 to 5 is $\div 4$.

Best Buys

Have a directly proportional relationship

To calculate best buys you need to be able to compare the cost of one unit or units of equal amounts



Shop A

4 cans for £1.20

$\downarrow$ £1.20 $\div$ 4

1 can is £0.30
Or 30p

Shop B

3 cans for 93p

$\downarrow$ £0.93 $\div$ 3

1 can is £0.31
Or 31p

Cost per item

Shop A is the best value as it is 1p cheaper per can of pop

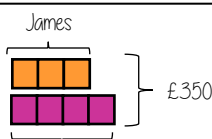
Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4.
 Work out how much each person earns

Model the Question

James: Lucy

3 : 4



Lucy

£350 $\div$ 7 = £50

□ = one part = £50

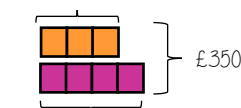
Find the value of one part

Whole: £350
 7 parts to share between
 (3 James, 4 Lucy)

Put back into the question

James: Lucy

James = 3 $\times$ £50 = £150



Lucy = 4 $\times$ £50 = £200

$\left(\begin{matrix} \times 50 & 3 : 4 & \times 50 \end{matrix} \right)$
 £150 : £200

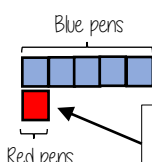
Finding a value given 1:n (or n:1)

Inside a box are blue and red pens in the ratio 5:1
 If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red
 5 : 1

□ = one part = 10 pens



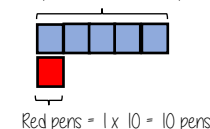
One unit = 10 pens

Put back into the question

Blue: Red

$\left(\begin{matrix} \times 10 & 5 : 1 & \times 10 \end{matrix} \right)$
 50 : 10

Blue pens = 5 $\times$ 10 = 50 pens



Red pens = 1 $\times$ 10 = 10 pens

There are 50 Blue Pens

Cost per item

Shop A is the best value as it is 1p cheaper per can of pop



Shop A

4 cans for £1.20

$\downarrow$ 4 $\div$ £1.20

Cost per pound

£1 buys 3.333 cans of pop

3 cans for 93p

$\downarrow$ 3 $\div$ £0.93

£1 buys 3.23 cans of pop

Shop A is still shown as being the best value but pay attention to the unit you are calculating, per item or per pound

Best value is the most product for the lowest price per unit

Year 9 Term 2

Linear Graphs

What do I need to be able to do?

By the end of this unit you should be able to:

- Compare gradients
- Compare intercepts
- Understand and use $y = mx + c$
- Find the equation of a line from a graph
- Interpret gradient and intercepts of real-life graphs

Keywords

Gradient: the steepness of a line

Intercept: where two lines cross. The y-intercept: where the line meets the y-axis

Parallel: two lines that never meet with the same gradient

Co-ordinate: a set of values that show an exact position on a graph

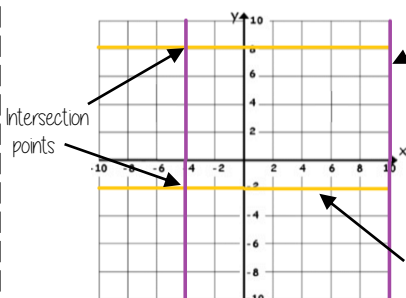
Linear: linear graphs (straight line) – linear common difference by addition/ subtraction

Asymptote: a straight line that a graph will never meet

Reciprocal: a pair of numbers that multiply together to give 1

Perpendicular: two lines that meet at a right angle

Lines parallel to the axes



All the points on this line have a x coordinate of 10

Lines parallel to the y axis take the form $x = a$ and are vertical

Lines parallel to the x axis take the form $y = a$ and are horizontal

All the points on this line have a y coordinate of -2

eg (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

'a' can be ANY positive or negative value including 0

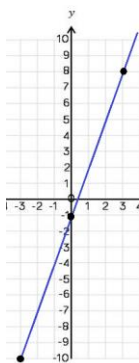
Plotting $y = mx + c$ graphs

$y = 3x - 1$ → 3 x the x coordinate then - 1

x	-3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

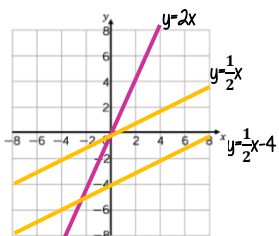
Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

Compare Gradients

$$y = mx + c$$

The coefficient of x (the number in front of x) tells us the gradient of the line



The greater the gradient – the steeper the line

Positive gradients

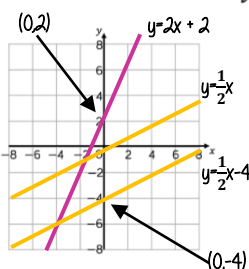
Negative gradients

Parallel lines have the same gradient

Compare Intercepts

$$y = mx + c$$

The value of c is the point at which the line crosses the y-axis Y intercept



The coordinate of a y intercept will always be (0, c)

Lines with the same y-intercept cross in the same place

$$y = mx + c$$

The coefficient of x (the number in front of x) tells us the gradient of the line

$$y = mx + c$$

The value of c is the point at which the line crosses the y-axis Y intercept

y and x are coordinates

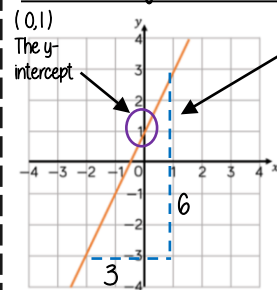
The equation of a line can be rearranged. Eg

$$y = c + mx$$

$$c = y - mx$$

Identify which coefficient you are identifying or comparing

Find the equation from a graph



The Gradient $\frac{6}{3} = 2$

$$y = 2x + 1$$

The direction of the line indicates a positive gradient

Positive gradients

Negative gradients

Real life graphs

A plumber charges a £25 callout fee, and then £12.50 for every hour. Complete the table of values to show the cost of hiring the plumber.

Time (h)	0	1	2	3	8
Cost (£)	£25				£125

In real life graphs like this values will always be positive because they measure distances or objects which cannot be negative.

Direct Proportion graphs

To represent direct proportion the graph must start at the origin

A box of pens costs £2.30

Complete the table of values to show the cost of buying boxes of pens.

Boxes	0	1	2	3	8
Cost (£)		£2.30			

When you have 0 pens this has 0 cost. The gradient shows the price per pen

The y-intercept shows the minimum charge. The gradient represents the price per mile