

Year 10 Foundation Term 2

Graphs

What do I need to be able to do?

By the end of this unit you should be able to:

- Compare gradients
- Compare intercepts
- Understand and use $y = mx + c$
- Find the equation of a line from a graph
- Interpret gradient and intercepts of real-life graphs

Keywords

Gradient: the steepness of a line

Intercept: where two lines cross. The y-intercept: where the line meets the y-axis.

Parallel: two lines that never meet with the same gradient.

Co-ordinate: a set of values that show an exact position on a graph

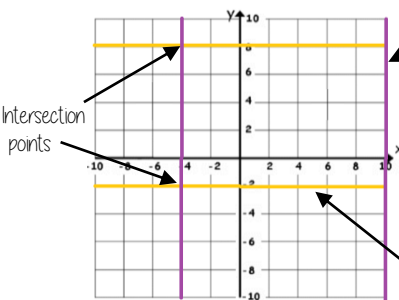
Linear: linear graphs (straight line) – linear common difference by addition/ subtraction

Asymptote: a straight line that a graph will never meet.

Reciprocal: a pair of numbers that multiply together to give 1

Perpendicular: two lines that meet at a right angle.

Lines parallel to the axes



All the points on this line have a x coordinate of 10

Lines parallel to the y axis take the form $x = a$ and are vertical

Lines parallel to the x axis take the form $y = a$ and are horizontal

All the points on this line have a y coordinate of -2

e.g. (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

'a' can be ANY positive or negative value including 0

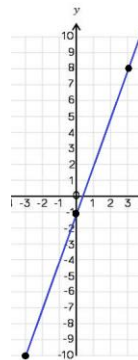
Plotting $y = mx + c$ graphs

$y = 3x - 1$ → 3 x the x coordinate then - 1

x	-3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

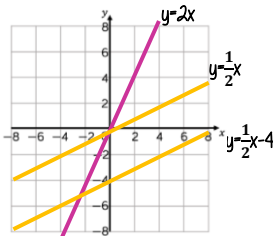
Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

Compare Gradients

$$y = mx + c$$

The coefficient of x (the number in front of x) tells us the gradient of the line



The greater the gradient – the steeper the line

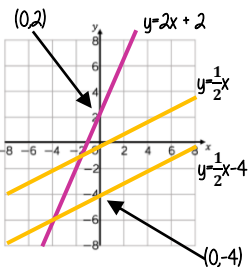
Parallel lines have the same gradient

Positive gradients
Negative gradients

Compare Intercepts

$$y = mx + c$$

The value of c is the point at which the line crosses the y-axis Y intercept



The coordinate of a y intercept will always be (0, c)

Lines with the same y-intercept cross in the same place

$$y = mx + c$$

The coefficient of x (the number in front of x) tells us the gradient of the line

$$y = mx + c$$

The value of c is the point at which the line crosses the y-axis Y intercept

y and x are coordinates

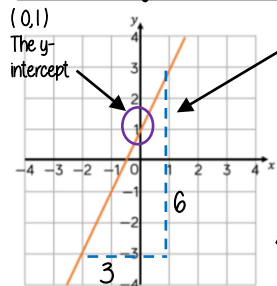
The equation of a line can be rearranged. E.g.

$$y = c + mx$$

$$c = y - mx$$

Identify which coefficient you are identifying or comparing

Find the equation from a graph



The Gradient $\frac{6}{3} = 2$

$$y = 2x + 1$$

The direction of the line indicates a positive gradient

Positive gradients
Negative gradients

Real life graphs

A plumber charges a £25 callout fee, and then £12.50 for every hour. Complete the table of values to show the cost of hiring the plumber.

Time (h)	0	1	2	3	8
Cost (£)	£25				£125

In real life graphs like this values will always be positive because they measure distances or objects which cannot be negative.

Direct Proportion graphs

To represent direct proportion the graph must start at the origin

A box of pens costs £2.30

Complete the table of values to show the cost of buying boxes of pens.

Boxes	0	1	2	3	8
Cost (£)		£2.30			

When you have 0 pens this has 0 cost. The gradient shows the price per pen.

The y-intercept shows the minimum charge. The gradient represents the price per mile

Year 10 Foundation Term 2

Fractions, Indices and Standard Form

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify square and cube numbers
- Calculate higher powers and roots
- Understand powers of 10 and standard form
- Know the addition and subtraction rule for indices
- Understand power zero and negative indices
- Calculate with numbers in standard form

Keywords

Standard (index) Form: A system of writing very big or very small numbers

Commutative: an operation is commutative if changing the order does not change the result

Base: The number that gets multiplied by a power

Power: The exponent — or the number that tells you how many times to use the number in multiplication

Exponent: The power — or the number that tells you how many times to use the number in multiplication

Indices: The power or the exponent

Negative: A value below zero.

Coefficient: The number used to multiply a variable

Square and cube numbers

Square numbers

1, 4, 9, 16...

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

$$(2 \times 2 \times 3) \times (2 \times 2 \times 3)$$

12 x 12

Prime factors can find square roots

$$\sqrt{144} = 12$$

Cube numbers

1, 8, 27, 64, 125...

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

$$(2 \times 3) \times (2 \times 3) \times (2 \times 3)$$

6 x 6 x 6

$$\sqrt[3]{216} = 6$$

Higher powers and roots

$$x^n$$

x — the base number

n — power
(number of times multiplied by itself)

$$\sqrt[n]{x}$$

Finding the nth root of any value

Other mental strategies for square roots

$$\begin{aligned}\sqrt{810000} &= \sqrt{81} \times \sqrt{10000} \\ &= 9 \times 100 \\ &= 900\end{aligned}$$

Standard form

Any number between 1 and less than 10

$$A \times 10^n$$

Any integer

$$\begin{aligned}0.001 &= 1 \times \frac{1}{1000} \\ &= 1 \times 10^{-3}\end{aligned}$$

10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^1	10^0	10^{-1}	10^{-2}	10^{-3}
10	1	0.1	0.01	0.001

Any value to the power 0 always = 1

Numbers in standard form with negative powers will be less than 1

$$3.2 \times 10^{-4} = 3.2 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = 0.00032$$

Example

$$\begin{aligned}3.2 \times 10^4 &= 3.2 \times 10 \times 10 \times 10 \times 10 \\ &= 32000\end{aligned}$$

Non-example

$$\begin{aligned}(0.8) \times 10^4 &= 8000 \\ 5.3 \times 10^{07} &= 53000000\end{aligned}$$

Negative powers do not indicate negative solutions

Addition/ Subtraction Laws

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

Zero and negative indices

$$x^0 = 1$$

Any number divided by itself = 1

$$\frac{a^6}{a^6} = a^6 \div a^6$$

$$= a^{6-6} = a^0 = 1$$

Negative indices do not indicate negative solutions

$$\begin{aligned}2^2 &= 4 \\ 2^1 &= 2 \\ 2^0 &= 1\end{aligned}$$

$$2^{-1} = \frac{1}{2}$$

$$2^{-2} = \frac{1}{4}$$

Looking at the sequence can help to understand negative powers

Powers of powers

$$(x^a)^b = x^{ab}$$

$$(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$$

The same base and power is repeated Use the addition law for indices

$$(2^3)^4 = 2^{12} \leftarrow a \times b = 3 \times 4 = 12$$

NOTICE the difference

$$(2x^3)^4 = 2x^3 \times 2x^3 \times 2x^3 \times 2x^3$$

The addition law applies ONLY to the powers. The integers still need to be multiplied

$$(2x^3)^4 = 16x^{12}$$

Standard form calculations

Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard form at the end

Method 1

$$\begin{aligned}6 \times 10^5 + 8 \times 10^5 &= 600000 + 800000 \\ &= 1400000 \\ &= 1.4 \times 10^6\end{aligned}$$

Multiplication and division

$$\begin{aligned}\frac{1.5 \times 10^5}{0.3 \times 10^3} &\leftarrow \text{Division questions can look like this} \\ (1.5 \times 10^5) \div (0.3 \times 10^3) \\ (15 \div 0.3) \times 10^{5-3} \\ &= 5 \times 10^2\end{aligned}$$

Method 2

$$\begin{aligned}&= (6 + 8) \times 10^5 \\ &= 14 \times 10^5 \\ &= 1.4 \times 10^1 \times 10^5 \\ &= 1.4 \times 10^6\end{aligned}$$

This is not the final answer

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations

Year 10 Foundation Term 2

Congruence and Similarity

What do I need to be able to do?

By the end of this unit you should be able to:

- Enlarge by a positive scale factor
- Enlarge by a fractional scale factor
- Identify similar shapes
- Work out missing sides and angles in similar shapes
- Use parallel lines to find missing angles
- Understand similarity and congruence

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)

Scale Factor: the multiplier of enlargement

Centre of enlargement: the point the shape is enlarged from

Similar: when one shape can become another with a reflection, rotation, enlargement or translation

Congruent: the same size and shape

Corresponding: items that appear in the same place in two similar situations

Parallel: straight lines that never meet (equal gradients)

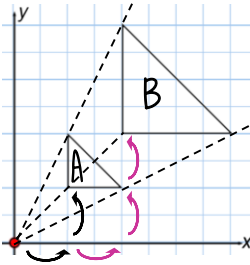
Positive scale factors R

Enlargement from a point

Enlarge shape A by SF 2 from (0,0)

The shape is enlarged by 2

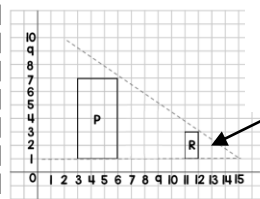
The distance from the point enlarges by 2



Fractional scale factors

Fractions less than 1 make a shape SMALLER

R is an enlargement of P by a scale factor $\frac{1}{3}$ from centre of enlargement (15,1)



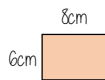
SF: $\frac{1}{3}$ - R is three times smaller than P

Identify similar shapes



Angles in similar shapes do not change.
e.g. if a triangle gets bigger the angles can not go above 180°

Similar shapes



Compare sides:

8cm

6cm

6 : 9

2 : 3

12cm

9cm

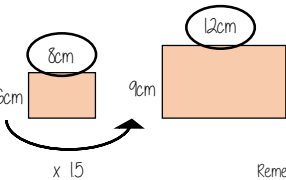
Scale Factor:
Both sides on the bigger shape are 1.5 times bigger

8 : 12

2 : 3

Both sets of sides are in the same ratio

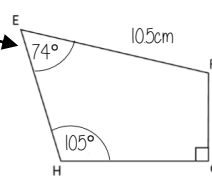
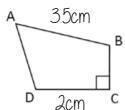
Information in similar shapes



Compare the equivalent side on both shapes

Scale Factor is the multiplicative relationship between the two lengths

Remember angles do not increase or change with scale



Shape ABCD and EFGH are similar

Notation helps us find the corresponding sides

AB and EF are corresponding

Angles in parallel lines

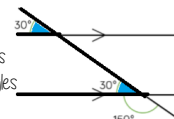
Alternate angles



Because alternate angles are equal the highlighted angles are the same size

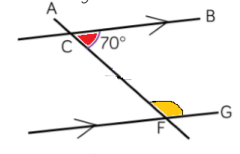
Corresponding angles

Because corresponding angles are equal the highlighted angles are the same size



Co-interior angles

Because co-interior angles have a sum of 180° the highlighted angle is 110°



As angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

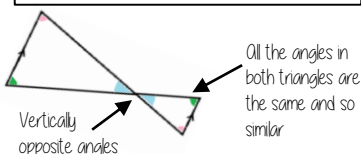
Similar triangles

Shares a vertex

Because corresponding angles are equal the highlighted angles are the same size

Parallel lines - all angles will be the same in both triangle

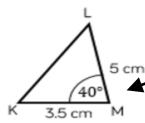
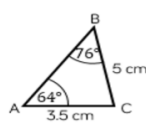
As all angles are the same this is similar - it only one pair of sides are needed to show equality



All the angles in both triangles are the same and so similar

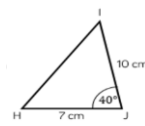
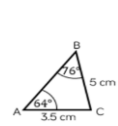
Congruence and Similarity

Congruent shapes are identical - all corresponding sides and angles are the same size



$\angle C \hat{B} A = \angle M \hat{K} L$

Because all the angles are the same and $AC = KM$ $BC = LM$ triangles ABC and KLM are **congruent**



Because all angles are the same, but all sides are enlarged by 2 ABC and HIJ are **similar**

Conditions for congruent triangles

Triangles are congruent if they satisfy any of the following conditions

Side-side-side

All three sides on the triangle are the same size

Angle-side-angle

Two angles and the side connecting them are equal in two triangles

Side-angle-side

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side

The triangles both have a right angle, the hypotenuse and one side are the same

Year 10 Foundation Term 2

Vectors

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand and represent vectors
- Use and read vector notation
- Draw and understand vectors multiplied by a scalar
- Draw and understand addition of vectors
- Draw and understand addition and subtraction of vectors

Keywords

Direction: the line our course something is going

Magnitude: the magnitude of a vector is its length

Scalar: a single number used to represent the multiplier when working with vectors

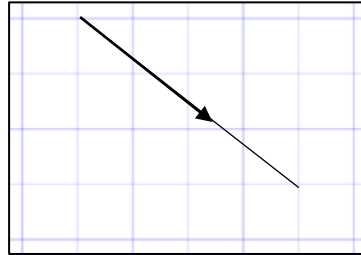
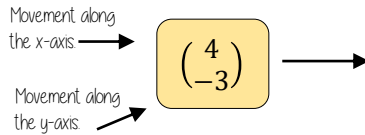
Column vector: a matrix of one column describing the movement from a point

Resultant: the vector that is the sum of two or more other vectors

Parallel: straight lines that never meet

Understand and represent vectors

Column vectors have been seen in translations to describe the movement of one image onto another



Vectors show both direction and magnitude

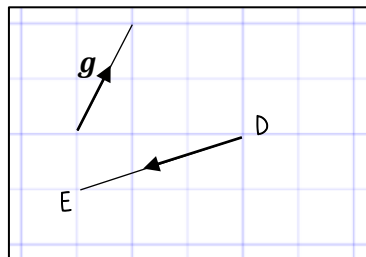
The arrow is pointing in the direction from starting point to end point of the vector.

The direction is important to correctly write the vector

The magnitude is the length of the vector (This is calculated using Pythagoras theorem and forming a right-angled triangle with auxiliary lines)

The magnitude stays the same even if the direction changes

Understand and represent vectors



Vector notation $\overrightarrow{DE}$ is another way to represent the vector joining the point D to the point E

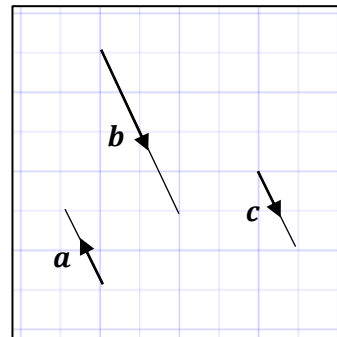
$$\overrightarrow{DE} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$$

The arrow also indicates the direction from point D to point E

Vectors can also be written in bold lower case so $\mathbf{g}$ represents the vector $\mathbf{g} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Vectors multiplied by a scalar

Parallel vectors are scalar multiples of each other



$$\mathbf{b} = 2 \times \mathbf{c} = 2\mathbf{c}$$

Multiply $\mathbf{c}$ by 2 this becomes $\mathbf{b}$.
The two lines are parallel

$$\mathbf{a} = -1 \times \mathbf{c} = -\mathbf{c}$$

The vectors $\mathbf{a}$ and $\mathbf{c}$ are also parallel. A negative scalar causes the vector to reverse direction

$$\mathbf{b} = -2 \times \mathbf{a} = -2\mathbf{a}$$

$$\mathbf{a} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

Addition of vectors

$$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\overrightarrow{BC} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

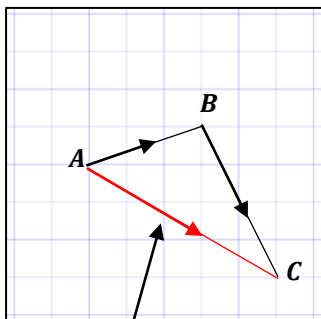
$$\overrightarrow{AB} + \overrightarrow{BC}$$

$$= \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} 3+2 \\ 1+(-4) \end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

Look how this addition compares to the vector $\overrightarrow{AC}$



The resultant

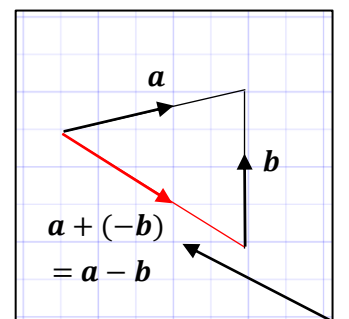
$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

Addition and subtraction of vectors

$$\mathbf{a} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$$\mathbf{b} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

$$\mathbf{a} + (-\mathbf{b}) = \begin{pmatrix} 5 & -0 \\ 1 & -4 \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$$



$$\mathbf{a} + (-\mathbf{b}) = \mathbf{a} - \mathbf{b}$$

The resultant is $\mathbf{a} - \mathbf{b}$ because the vector is in the opposite direction to $\mathbf{b}$ which needs a scalar of -1

Year 10 Foundation Term 2

More Algebra

What do I need to be able to do?

By the end of this unit you should be able to:

- Determine whether (x,y) is a solution
- Solve by substituting a known variable
- Solve by substituting an expression
- Solve graphically
- Solve by subtracting/ adding equations
- Solve by adjusting equations
- Form and solve linear simultaneous equations

Keywords

Solution: a value we can put in place of a variable that makes the equation true

Variable: a symbol for a number we don't know yet

Equation: an equation says that two things are equal – it will have an equals sign =

Substitute: replace a variable with a numerical value

LCM: lowest common multiple (the first time the times table of two or more numbers match)

Eliminate: to remove

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

Coordinate: a set of values that show an exact position

Intersection: the point two lines cross or meet

Is (x, y) a solution?

x and y represent values that can be substituted into an equation

Does the coordinate (1,8) lie on the line $y=3x+5$?

This coordinate represents $x=1$ and $y=8$

$$y = 3x + 5$$

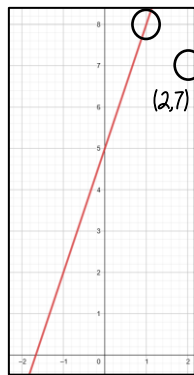
$$8 = 3(1) + 5$$

As the substitution makes the equation correct the coordinate (1,8) IS on the line $y=3x+5$

Is (2,7) on the same line?

$$7 \neq 3(2) + 5$$

No 7 does NOT equal $6+5$



Substituting known variables

A line has the equation $3x + y = 14$

Two different variables, two solutions

Stephanie knows the point $x = 4$ lies on that line. Find the value for y

$$x = 4$$

$$3x + y = 14$$

$$3(4) + y = 14$$

$$12 + y = 14$$

$$-12 \quad -12$$

$$y = 2$$

Substituting in an expression

$$x = 2y$$

$$x + y = 30$$

Pair of simultaneous equations (two representations)

Substitute $2y$ in place of the x variable as they represent the same value

$$x = 2y$$

$$x + y = 30$$

$$3y = 30$$

$$3y = 30$$

$$\div 3$$

$$\div 3$$

$$y = 10$$

$$x = 2y$$

$$10 \quad 10$$

$$x = 20$$

Solve graphically

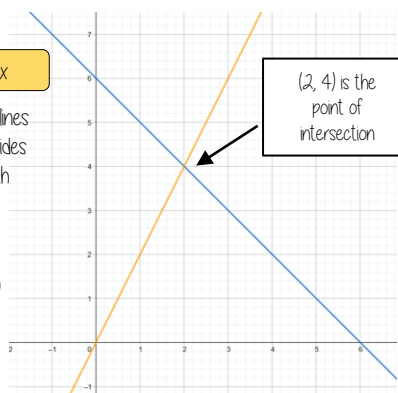
$$x + y = 6$$

$$y = 2x$$

Linear equations are straight lines
The point of intersection provides the x and y solution for both equations

The solution that satisfies both equations is

$$x = 2 \text{ and } y = 4$$



Solve by subtraction

$$18$$

$$10$$

$$8$$

$$x = 4$$

$$y = 3$$

$$3x + 2y = 18$$

$$x + 2y = 10$$

$$2x = 8$$

$$\div 2 \quad \div 2$$

$$x = 4$$

$$x + 2y = 10$$

$$(4) + 2y = 10$$

$$-4 \quad -4$$

$$2y = 6$$

$$\div 2 \quad \div 2$$

$$y = 3$$

$$x + x + x + y + y = 18$$

$$x + y + y = 10$$

$$x + x + y + y = 18$$

$$y + y = 10$$

$$x + x = 8$$

$$x = 4$$

$$y = 3$$

Solve by addition

Addition makes zero pairs

$$3x + 2y = 16$$

$$+ 6x - 2y = 2$$

$$9x = 18$$

$$\div 9 \quad \div 9$$

$$x = 2$$

$$3x + 2y = 16$$

$$3(2) + 2(y) = 16$$

$$6 + 2y = 16$$

$$-6 \quad -6$$

$$2y = 10$$

$$y = 5$$

$$x + x + x = 18$$

$$x + x + x = 18$$

$$x + x + x = 18$$

$$x + x + x = 18$$

$$x + x + x = 18$$

$$x + x + x = 18$$

$$x + x + x = 18$$

$$x + x + x = 18$$

Solve by adjusting one

$$h + j = 12$$

$$2h + 2j = 29$$

$$2h + 2j = 24$$

$$2h + 2j = 29$$

By proportionally adjusting one of the equations – now solve the simultaneous equations choosing an addition or subtraction method

$$12$$

$$29$$

$$24$$

$$29$$

$$29$$

$$29$$

$$29$$

$$29$$

$$29$$

$$29$$

$$29$$

Solve by adjusting both

$$2x + 3y = 39$$

$$5x - 2y = -7$$

Use LCM to make equivalent x OR y values

Because of the negative values using zero pairs and y values is chosen choice

$$4x + 6y = 78$$

$$15x - 6y = -21$$

Now solve by addition

$$x + x + x + y + y + y = 39$$

$$x + x + x + y + y + y = 39$$

$$x + x + x + y + y + y = 39$$

$$x + x + x + y + y + y = 39$$

$$x + x + x + y + y + y = 39$$

$$x + x + x + y + y + y = 39$$