

Year 10 Foundation Term 3

More Algebra

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve inequalities with negative numbers
- Solve equations with unknowns on both sides
- Solve inequalities with unknowns on both sides
- Substitute into formulae and equations
- Rearrange formulae

Keywords

Inequality: an inequality compares two values showing if one is greater than, less than or equal to another

Variable: a quantity that may change within the context of the problem

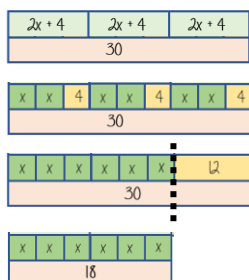
Rearrange: Change the order

Inverse operation: the operation that reverses the action

Substitute: replace a variable with a numerical value

Solve: find a numerical value that satisfies an equation

Solve equations with brackets



$$3(2x + 4) = 30$$

Expand the brackets

$$6x + 12 = 30$$

$$-12 \quad -12$$

$$6x = 18$$

$$-6 \quad -6$$

$$x = 3$$

Form and solve inequalities



Two more than treble my number is greater than 11

Find the possible range of values

$$3x + 2 > 11$$

Solve

$$x \leftarrow -3 \leftarrow -2 \leftarrow 11$$

$$x > 3$$

Inequalities with negatives

Method 1 Make x positive first

$$2 - 3x > 17$$

$$+3x \quad +3x$$

$$2 > 17 + 3x$$

$$-17 \quad -17$$

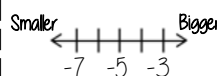
$$-15 > 3x$$

$$\div 3 \quad \div 3$$

$$-5 > x$$

x is true for any value smaller than -5

CHECK IT!
 $2 - 3(-6) = 20$
 TRUE/ CORRECT



Equations with unknown on both sides

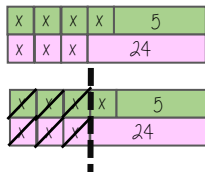
$$4x + 5 = 3x + 24$$

$$-3x \quad -3x$$

$$x + 5 = 24$$

$$-5 \quad -5$$

$$x = 19$$



Inequalities with unknown on both sides

Solving inequalities has the same method as equations

$$5(x + 4) < 3(x + 2)$$

$$5x + 20 < 3x + 6$$

$$2x + 20 < 6$$

$$2x < -14$$

$$x < -7$$

Check it!

$$5(-8 + 4) < 3(-8 + 2)$$

$$5(-4) < 3(-6)$$

$$-20 < -18$$

✓ -20 IS smaller than -18

Method 2 Keep the negative x

$$2 - 3x > 17$$

$$-2 \quad -2$$

$$-3x > 15$$

$$\div -3 \quad \div -3$$

$$x > -5$$

x is true for any value bigger than -5

This cannot be true...

$$x < -5$$

When you multiply or divide x by a negative you need to reverse the inequality

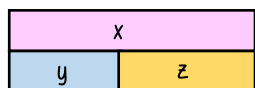
Formulae and Equations

Formulae — all expressed in symbols

Substitute in values

Equations — include numbers and can be solved

Rearranging Formulae (one step)



$$x = y + z$$

Rearrange to make y the subject

$$y = x - z$$

$$y \rightarrow +z \rightarrow x$$

$$y \leftarrow -z \leftarrow x$$

Using inverse operations or fact families will guide you through rearranging formulae

Rearranging can also be checked by substitution

Language of rearranging...

Make XXX the subject

Change the subject

Rearrange

Rearranging Formulae (two step)

In an equation (find x)

$$4x - 3 = 9$$

$$+3 \quad +3$$

$$4x = 12$$

$$\div 4 \quad \div 4$$

$$x = 3$$

In a formula (make x the subject)

$$xy - s = a$$

$$+s \quad +s$$

$$xy = a + s$$

$$\div y \quad \div y$$

$$x = \frac{a + s}{y}$$

The steps are the same for solving and rearranging

Rearranging is often needed when using $y = mx + c$

e.g Find the gradient of the line $2y - 4x = 9$

Make y the subject first $y = \frac{4x + 9}{2}$

Gradient = $\frac{4}{2} = 2$

Year 10 Foundation Term 3

Perimeter, Area and Volume

What do I need to be able to do?

By the end of this unit you should be able to:

- Name 2D & 3D shapes
- Recognise Prisms
- Sketch and recognise nets
- Draw plans and elevations
- Find areas of 2D shapes
- Find Surface area for cubes, cuboids, triangular prisms and cylinders
- Find the volume of 3D shapes

Keywords

2D: two dimensions to the shape e.g length and width

3D: three dimensions to the shape e.g length, width and height

Vertex: a point where two or more line segments meet

Edge: a line on the boundary joining two vertex

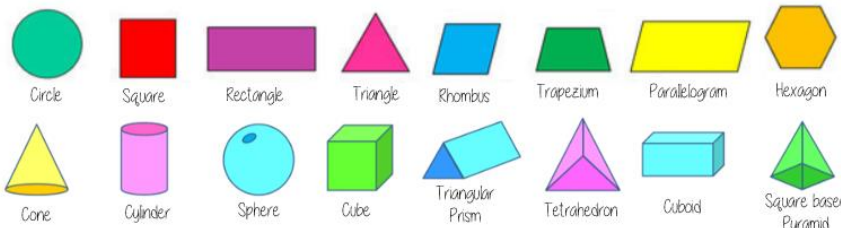
Face: a flat surface on a solid object

Cross-section: a view inside a solid shape made by cutting through it

Plan: a drawing of something when drawn from above (sometimes birds eye view)

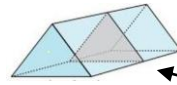
Perspective: a way to give illustration of a 3D shape when drawn on a flat surface.

Name 2D & 3D shapes



Recognise prisms

A solid object with two identical ends and flat sides

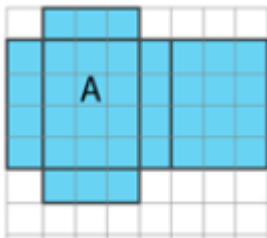
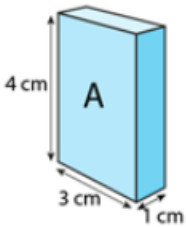


The cross section will also be identical to the end faces



A cylinder although with very similar properties does not have flat faces so is not categorised as a prism

Nets of cuboids



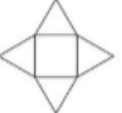
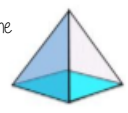
1cm grids help to draw accurately

Visualise the folding of the net. Will it make the cuboid with all sides touching

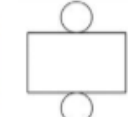
Sketch and recognise nets



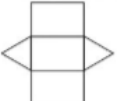
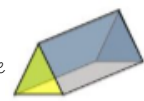
Do they have the same number of faces?



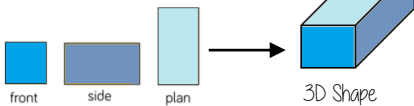
Where do the edges join?



Are the shapes of the faces correct?



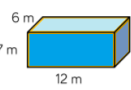
Plans and elevations



The direction you are considering the shape from determines the front and side views

Surface area

Sketching nets first helps you visualise all the sides that will form the overall surface area

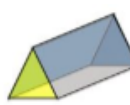


For cubes and cuboids you can also find one of each face and double it



Sides 6×7
 6×7
 Front and back 12×7
 12×7
 Top and Bottom 12×6
 12×6

Sum of all sides is surface area



For other shapes - not all the sides are the same, so calculate the individually

Volumes

Volume is the 3D space it takes up - also known as capacity if using liquids to fill the space



Counting cubes

Some 3D shape volumes can be calculated by counting the number of cubes that fit inside the shape

Cubes/ Cuboids = base x width x height

Remember multiplication is commutative



Cross section



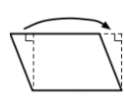
Prisms and cylinders = area cross section x height

Height can also be described as depth

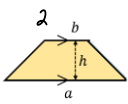
Area of 2D shapes



Parallelogram/ Rhombus
 Base x Perpendicular height



Area of a trapezium
 $(a + b) \times h$



Area of a circle
 $\pi \times \text{radius}^2$



Surface area - cylinders



The area of the circle
 $\pi \times \text{radius}^2$

The width of this face is the same as the circumference
 $\pi \times \text{diameter} \times \text{height}$

$2 \times \pi \times \text{radius}^2 + \pi \times \text{diameter} \times \text{height}$

Areas - square units
 Volumes - cube units

Areas and volumes can be left in terms of π

Year 10 Foundation Term 3

Perimeter, Area and Volume

What do I need to be able to do?

By the end of this unit you should be able to:

- Recognise and label parts of a circle
- Calculate fractional parts of a circle
- Calculate the length of an arc
- Calculate the area of a sector
- Understand and use volume of a cone, cylinder and sphere.
- Understand and use surface area of a cone, cylinder and sphere.

Keywords

Circumference: the length around the outside of the circle – the perimeter

Area: the size of the 2D surface

Diameter: the distance from one side of a circle to another through the centre

Radius: the distance from the centre to the circumference of the circle

Tangent: a straight line that touches the circumference of a circle

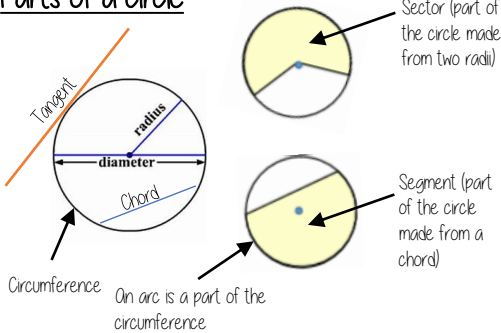
Chord: a line segment connecting two points on the curve

Frustum: a pyramid or cone with the top cut off

Hemisphere: half a sphere

Surface area: the total area of the surface of a 3D shape.

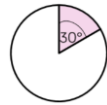
Parts of a circle



Fractional parts of a circle

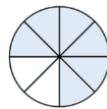
A circle is made up of 360°

Formula to remember:
Area of a circle = πr^2
Circumference of a circle = πd or $2\pi r$



30° represents $\frac{30}{360}$ of a full circle

$$\frac{30}{360} = \frac{1}{12}$$



$\frac{270}{360}$ of a full circle (in degrees)

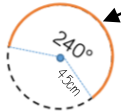
$\frac{6}{8}$ of a full circle (in equal parts)

$\frac{3}{4}$ of a full circle

The fraction of the circle is as $\frac{\theta}{360}$
 θ represents the degrees in the sector

Arc length

Remember an arc is part of the circumference
Circumference of the whole circle = $\pi d = \pi \times 9 = 9\pi$



$$\text{Arc length} = \frac{\theta}{360} \times \text{circumference}$$

$$= \frac{240}{360} \times 9\pi = \frac{2}{3} \times 9\pi = 6\pi$$

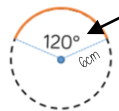
Perimeter

Perimeter is the length around the outside of the shape
This includes the arc length and the radii that enclose the shape

$$\text{Perimeter} = \frac{\theta}{360} \times \text{circumference} + 2r = 6\pi + 9$$

Sector area

Remember a sector is part of a circle
Area of the whole circle = $\pi r^2 = \pi \times 6^2 = 36\pi$



$$\text{Sector area} = \frac{\theta}{360} \times \text{area of circle}$$

$$= \frac{120}{360} \times 36\pi = \frac{1}{3} \times 36\pi = 12\pi$$

Volume of a cone and a cylinder

$$\text{Volume Cylinder} = \pi r^2 h$$

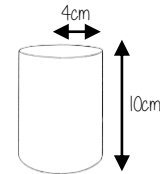


A cylinder is a prism – cross section is a circle



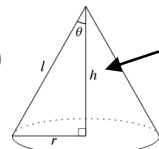
A cone is a pyramid with a circular base

$$\text{Volume Cone} = \frac{1}{3} \pi r^2 h$$



$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times 4^2 \times 10 \\ &= \pi \times 160 \\ &= 160\pi \text{ cm}^2 \end{aligned}$$

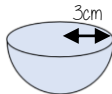
Give your answer in terms of π'
means NOT in terms of pi $\approx 502.7 \text{ cm}^2$



The height of a cone is the perpendicular height from the vertex to the base

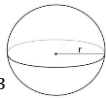
Look out for trigonometry or Pythagoras theorem – the radius forms the base of a right-angled triangle

Volume of a sphere



$$\begin{aligned} \text{Volume Sphere} &= \frac{4}{3} \pi r^3 \\ &= \frac{4}{3} \times \pi \times 3^3 \\ &= \frac{4}{3} \times \pi \times 27 = 36\pi \end{aligned}$$

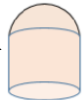
A hemisphere is half the volume of the overall sphere
 $= 36\pi \div 2 = 18\pi$



$$\text{Volume Sphere} = \frac{4}{3} \pi r^3$$

NOTE: This is now a cubed value

Look out for hemispheres being placed on other 3D shapes, e.g. cones and cylinders



Surface area of a sphere



Radius = 5cm

$$\text{Surface area} = 4\pi r^2$$

$$= 4 \times \pi \times 5^2$$

$$= 4 \times \pi \times 25$$

The curved surface area of a sphere

$$= 100\pi$$

$$\text{Surface area} = 4\pi r^2$$

A hemisphere has the curved surface AND a flat circular face



$$= 100\pi \div 2 = 50\pi$$

$$= 50\pi + \pi \times 5^2$$

$$\text{Hemisphere} = 75\pi$$

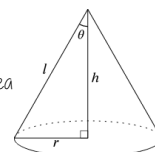
Surface area of cones and cylinders

$$\text{Surface area cylinder} = 2\pi r^2 + \pi dh$$



The area of two circles (top and bottom face) + the area of the curved face

The length of shape B is the circumference of the circles



$$\text{Curved surface area Cone} = \pi r l$$

Look out for the use of Pythagoras to calculate the length l

Total surface area = curved face + circle face (area of base)

Year 10 Foundation Term 3

Transformations

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify the order of rotational symmetry
- Rotate a shape about a point on the shape
- Rotate a shape about a point not on a shape
- Translate by a given vector
- Compare rotations and reflections

Keywords

Rotate: a rotation is a circular movement

Symmetry: when two or more parts are identical after a transformation

Regular: a regular shape has angles and sides of equal lengths

Invariant: a point that does not move after a transformation

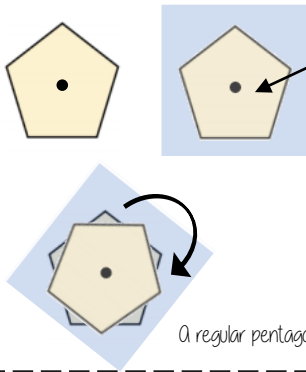
Vertex: a point two edges meet

Horizontal: from side to side

Vertical: from up to down

Rotational Symmetry

Tracing paper helps check rotational symmetry



1 Trace your shape (mark the centre point)

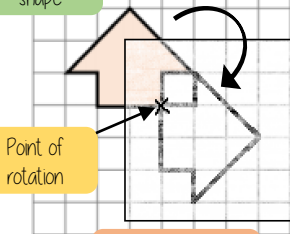
2 Rotate your tracing paper on top of the original through 360°

3 Count the times it fits back into itself

A regular pentagon has rotational symmetry of order 5

Rotate from a point (in a shape)

Original shape



Point of rotation

Image: 90° clockwise

1 Trace the original shape (mark the point of rotation)

2 Keep the point in the same place and turn the tracing paper

3 Draw the new shape



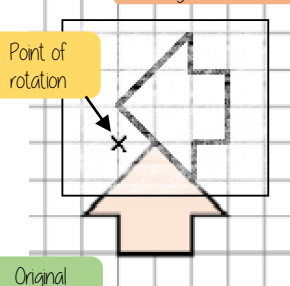
Clockwise



Anti-Clockwise

Rotate from a point (outside a shape)

Image: 90° anti-clockwise



Point of rotation

Original shape

1 Trace the original shape (mark the point of rotation)

2 Keep the point in the same place and turn the tracing paper

3 Draw the new shape

Translation and vector notation

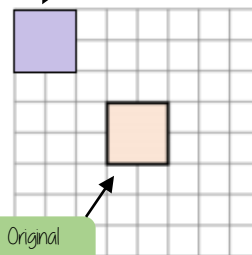
Vector Notation

$$\begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

How far left or right to move
Negative value (left)
Positive value (right)

How far up or down to move
Negative value (down)
Positive value (up)

Translation $\begin{pmatrix} -3 \\ 3 \end{pmatrix}$

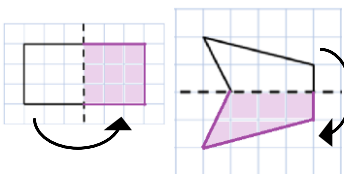


Original shape

Every vertex has been translated by the same amount

Compare rotations and reflections

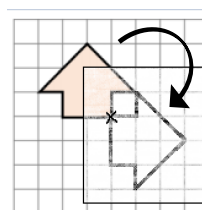
Reflections are a mirror image of the original shape.



Information needed to perform a reflection:

- Line of reflection (Mirror line)

Rotations are the movement of a shape in a circular motion



Information needed to perform a rotation:

- Point of rotation
- Direction of rotation
- Degrees of rotation