

Year 11 Foundation Term 1

Perimeter, Area and Volume

What do I need to be able to do?

By the end of this unit you should be able to:

- Recognise and label parts of a circle
- Calculate fractional parts of a circle
- Calculate the length of an arc
- Calculate the area of a sector
- Understand and use volume of a cone, cylinder and sphere.
- Understand and use surface area of a cone, cylinder and sphere.

Keywords

Circumference: the length around the outside of the circle – the perimeter

Area: the size of the 2D surface

Diameter: the distance from one side of a circle to another through the centre

Radius: the distance from the centre to the circumference of the circle

Tangent: a straight line that touches the circumference of a circle

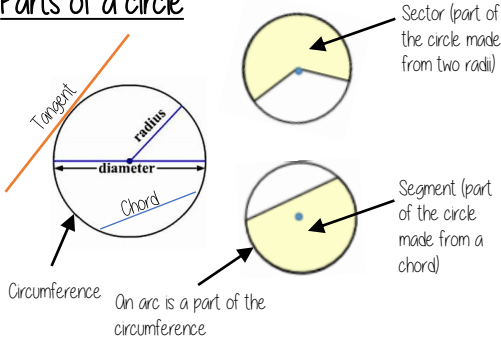
Chord: a line segment connecting two points on the curve

Frustum: a pyramid or cone with the top cut off

Hemisphere: half a sphere

Surface area: the total area of the surface of a 3D shape.

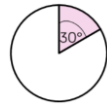
Parts of a circle



Fractional parts of a circle

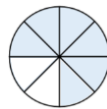
A circle is made up of 360°

Formula to remember:
Area of a circle = πr^2
Circumference of a circle = πd or $2\pi r$



30° represents $\frac{30}{360}$ of a full circle

$$\frac{30}{360} = \frac{1}{12}$$



$\frac{270}{360}$ of a full circle (in degrees)

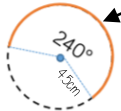
$\frac{6}{8}$ of a full circle (in equal parts)

$\frac{3}{4}$ of a full circle

The fraction of the circle is as $\frac{\theta}{360}$
 θ represents the degrees in the sector

Arc length

Remember an arc is part of the circumference
Circumference of the whole circle = $\pi d = \pi \times 9 = 9\pi$



$$\text{Arc length} = \frac{\theta}{360} \times \text{circumference}$$

$$= \frac{240}{360} \times 9\pi$$

$$= \frac{2}{3} \times 9\pi = 6\pi$$

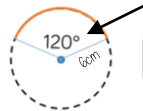
Perimeter

Perimeter is the length around the outside of the shape
This includes the arc length and the radii that enclose the shape

$$\text{Perimeter} = \frac{\theta}{360} \times \text{circumference} + 2r = 6\pi + 9$$

Sector area

Remember a sector is part of a circle
Area of the whole circle = $\pi r^2 = \pi \times 6^2 = 36\pi$



$$\text{Sector area} = \frac{\theta}{360} \times \text{area of circle}$$

$$= \frac{120}{360} \times 36\pi$$

$$= \frac{1}{3} \times 36\pi = 12\pi$$

Volume of a cone and a cylinder

$$\text{Volume Cylinder} = \pi r^2 h$$

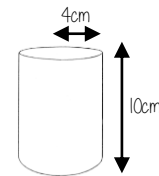


A cylinder is a prism – cross section is a circle



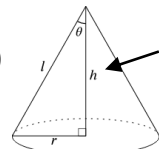
$$\text{Volume Cone} = \frac{1}{3} \pi r^2 h$$

A cone is a pyramid with a circular base



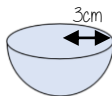
$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times 4^2 \times 10 \\ &= \pi \times 160 \\ &= 160\pi \text{ cm}^2 \end{aligned}$$

Give your answer in terms of π'
means NOT in terms of pi $\approx 502.7 \text{ cm}^2$



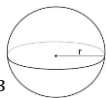
The height of a cone is the perpendicular height from the vertex to the base
Look out for trigonometry or Pythagoras theorem – the radius forms the base of a right-angled triangle

Volume of a sphere



$$\begin{aligned} \text{Volume Sphere} &= \frac{4}{3} \pi r^3 \\ &= \frac{4}{3} \times \pi \times 3^3 \\ &= \frac{4}{3} \times \pi \times 27 = 36\pi \end{aligned}$$

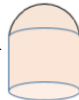
A hemisphere is half the volume of the overall sphere
 $= 36\pi \div 2 = 18\pi$



$$\text{Volume Sphere} = \frac{4}{3} \pi r^3$$

NOTE: This is now a cubed value

Look out for hemispheres being placed on other 3D shapes, e.g. cones and cylinders



Surface area of a sphere



Radius = 5cm

$$\text{Surface area} = 4\pi r^2$$

$$= 4 \times \pi \times 5^2$$

$$= 4 \times \pi \times 25$$

The curved surface area of a sphere

$$= 100\pi$$

$$\text{Surface area} = 4\pi r^2$$

A hemisphere has the curved surface AND a flat circular face



$$= 100\pi \div 2 = 50\pi$$

$$= 50\pi + \pi \times 5^2$$

$$\text{Hemisphere} = 75\pi$$

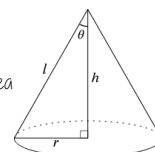
Surface area of cones and cylinders

$$\text{Surface area cylinder} = 2\pi r^2 + \pi d h$$



The area of two circles (top and bottom face) + the area of the curved face

The length of shape B is the circumference of the circles



$$\text{Curved surface area Cone} = \pi r l$$

Look out for the use of Pythagoras to calculate the length l

Total surface area = curved face + circle face (area of base)

Year 11 Foundation Term 1

Perimeter, Area and Volume

What do I need to be able to do?

By the end of this unit you should be able to:

- Name 2D & 3D shapes
- Recognise Prisms
- Sketch and recognise nets
- Draw plans and elevations
- Find areas of 2D shapes
- Find Surface area for cubes, cuboids, triangular prisms and cylinders
- Find the volume of 3D shapes

Keywords

2D: two dimensions to the shape e.g length and width

3D: three dimensions to the shape e.g length, width and height

Vertex: a point where two or more line segments meet

Edge: a line on the boundary joining two vertex

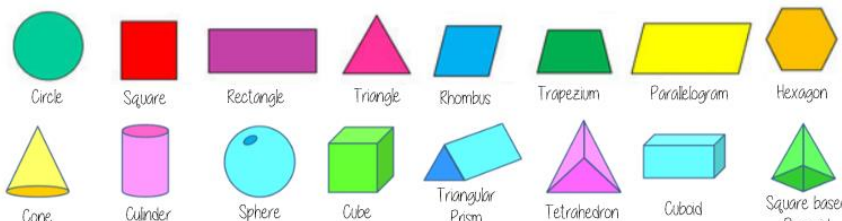
Face: a flat surface on a solid object

Cross-section: a view inside a solid shape made by cutting through it

Plan: a drawing of something when drawn from above (sometimes birds eye view)

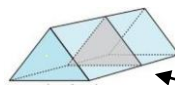
Perspective: a way to give illustration of a 3D shape when drawn on a flat surface.

Name 2D & 3D shapes



Recognise prisms

A solid object with two identical ends and flat sides

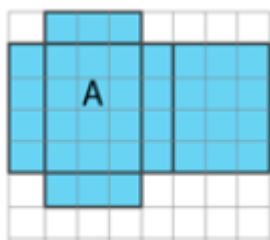
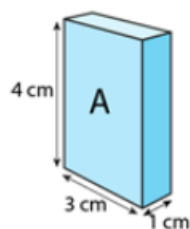


The cross section will also be identical to the end faces



A cylinder although with very similar properties does not have flat faces so is not categorised as a prism

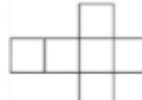
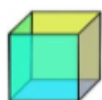
Nets of cuboids



1cm grids help to draw accurately

Visualise the folding of the net. Will it make the cuboid with all sides touching

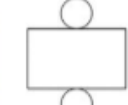
Sketch and recognise nets



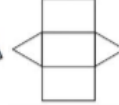
Do they have the same number of faces?



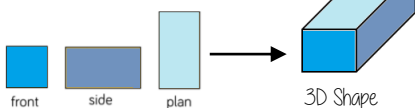
Where do the edges join?



Are the shapes of the faces correct?



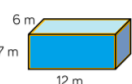
Plans and elevations



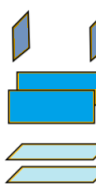
The direction you are considering the shape from determines the front and side views

Surface area

Sketching nets first helps you visualise all the sides that will form the overall surface area

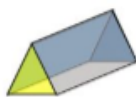


For cubes and cuboids you can also find one of each face and double it



Sides 6×7
 6×7
 Front and back 12×7
 12×7
 Top and Bottom 12×6
 12×6

Sum of all sides is surface area



For other shapes - not all the sides are the same, so calculate the individually

Area of 2D shapes

Rectangle $\text{Base} \times \text{Height}$



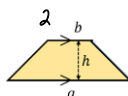
Triangle $\frac{1}{2} \times \text{Base} \times \text{Perpendicular height}$



Parallelogram/ Rhombus $\text{Base} \times \text{Perpendicular height}$



Area of a trapezium $\frac{(a+b) \times h}{2}$



Area of a circle $\pi \times \text{radius}^2$



Surface area - cylinders



The area of the circle $\pi \times \text{radius}^2$

The width of this face is the same as the circumference $\pi \times \text{diameter} \times \text{height}$

$$2 \times \pi \times \text{radius}^2 + \pi \times \text{diameter} \times \text{height}$$

Volumes

Volume is the 3D space it takes up - also known as capacity if using liquids to fill the space



Counting cubes

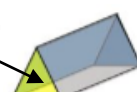
Some 3D shape volumes can be calculated by counting the number of cubes that fit inside the shape

$$\text{Cubes/ Cuboids} = \text{base} \times \text{width} \times \text{height}$$

Remember multiplication is commutative



Cross section



Cross section

$$\text{Prisms and cylinders} = \text{area cross section} \times \text{height}$$

Height can also be described as depth

Areas - square units
 Volumes - cube units

Areas and volumes can be left in terms of π

Year 11 Foundation Term 1

Laws of Indices and Standard Form

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify square and cube numbers
- Calculate higher powers and roots
- Understand powers of 10 and standard form
- Know the addition and subtraction rule for indices
- Understand power zero and negative indices
- Calculate with numbers in standard form

Keywords

Standard (index) Form: A system of writing very big or very small numbers

Commutative: an operation is commutative if changing the order does not change the result

Base: The number that gets multiplied by a power

Power: The exponent — or the number that tells you how many times to use the number in multiplication

Exponent: The power — or the number that tells you how many times to use the number in multiplication

Indices: The power or the exponent

Negative: A value below zero.

Coefficient: The number used to multiply a variable

Square and cube numbers

Square numbers

1, 4, 9, 16...

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

$$(2 \times 2 \times 3) \times (2 \times 2 \times 3)$$

12 x 12

Prime factors can find square roots

$$\sqrt{144} = 12$$

Cube numbers

1, 8, 27, 64, 125...

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

$$(2 \times 3) \times (2 \times 3) \times (2 \times 3)$$

6 x 6 x 6

$$\sqrt[3]{216} = 6$$

Higher powers and roots

$$x^n$$

x — the base number

n — power
(number of times multiplied by itself)

$$\sqrt[n]{x}$$

Finding the nth root of any value

Other mental strategies for square roots

$$\begin{aligned}\sqrt{810000} &= \sqrt{81} \times \sqrt{10000} \\ &= 9 \times 100 \\ &= 900\end{aligned}$$

Standard form

Any number between 1 and less than 10

$$A \times 10^n$$

Any integer

$$\begin{aligned}0.001 &= 1 \times \frac{1}{1000} \\ &= 1 \times 10^{-3}\end{aligned}$$

10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^1	10^0	10^{-1}	10^{-2}	10^{-3}
10	1	0.1	0.01	0.001

Any value to the power 0 always = 1

Numbers in standard form with negative powers will be less than 1

$$3.2 \times 10^{-4} = 3.2 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = 0.00032$$

Example

$$\begin{aligned}3.2 \times 10^4 &= 3.2 \times 10 \times 10 \times 10 \times 10 \\ &= 32000\end{aligned}$$

Non-example

$$\begin{aligned}(0.8) \times 10^4 &= 0.8 \times 10000 \\ &= 8000\end{aligned}$$

Negative powers do not indicate negative solutions

Addition/ Subtraction Laws

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

Zero and negative indices

$$x^0 = 1$$

$$\begin{aligned}\frac{a^6}{a^6} &= a^6 \div a^6 \\ &= a^{6-6} = a^0 = 1\end{aligned}$$

Negative indices do not indicate negative solutions

$$\begin{aligned}2^2 &= 4 \\ 2^1 &= 2 \\ 2^0 &= 1\end{aligned}$$

$$2^{-1} = \frac{1}{2}$$

$$2^{-2} = \frac{1}{4}$$

Looking at the sequence can help to understand negative powers

Powers of powers

$$(x^a)^b = x^{ab}$$

$$(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$$

The same base and power is repeated Use the addition law for indices

$$(2^3)^4 = 2^{12} \leftarrow a \times b = 3 \times 4 = 12$$

NOTICE the difference

$$(2x^3)^4 = 2x^3 \times 2x^3 \times 2x^3 \times 2x^3$$

The addition law applies ONLY to the powers. The integers still need to be multiplied

$$(2x^3)^4 = 16x^{12}$$

Standard form calculations

Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard form at the end

Method 1

$$\begin{aligned}6 \times 10^5 + 8 \times 10^5 &= 600000 + 800000 \\ &= 1400000 \\ &= 1.4 \times 10^6\end{aligned}$$

Multiplication and division

$$\begin{aligned}\frac{1.5 \times 10^5}{0.3 \times 10^3} &= \frac{1.5}{0.3} \times \frac{10^5}{10^3} \\ &= (1.5 \div 0.3) \times 10^{5-3} \\ &= 5 \times 10^2\end{aligned}$$

Method 2

$$\begin{aligned}(6 + 8) \times 10^5 &= 14 \times 10^5 \\ &= 1.4 \times 10^1 \times 10^5 \\ &= 1.4 \times 10^6\end{aligned}$$

This is not the final answer

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations

Year 11 Foundation Term 1

Congruence and Similarity

What do I need to be able to do?

By the end of this unit you should be able to:

- Recognise enlargement and similarity
- Enlarge a shape by a positive SF
- Enlarge a shape from a point
- Enlarge a shape by a fractional SF
- Work out missing sides and angles in a pair of similar shapes

Keywords

Similar Shapes: shapes of different sizes that have corresponding sides in equal proportion and identical corresponding angles.

Scale Factor: the multiple describing how much a shape has been enlarged

Enlarge: to change the size of a shape (enlargement is not always making a shape bigger)

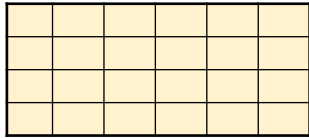
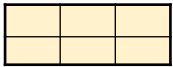
Corresponding: objects (or sides) that appear in the same place in two similar situations.

Image: the picture or visual representation of the shape

Recognise enlargement & similarity

Shapes are similar if all pairs of corresponding sides are in the same ratio

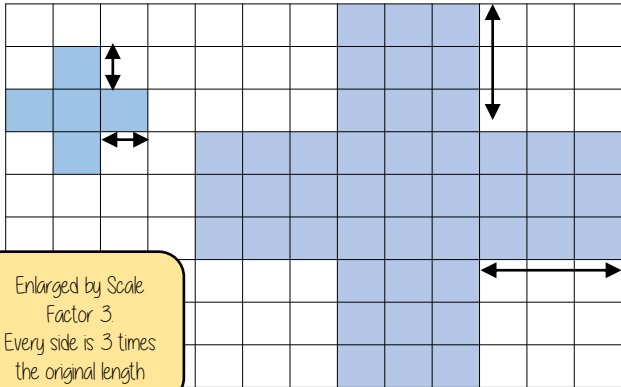
These shapes are similar because all sides are increased by the same ratio



Enlargements are similar shapes with a ratio other than 1

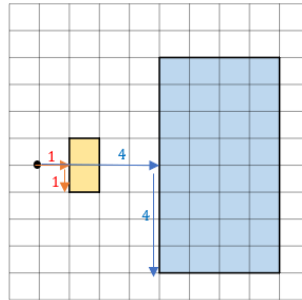
Enlarge by a positive scale factor

With a scale factor larger than 1 it makes the shape bigger



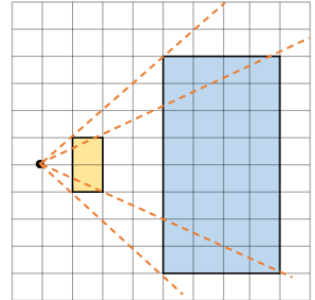
Enlarge a shape from a point

Scaled distances method



Scale the distance between the point of enlargement and each corresponding vertices

Rays method

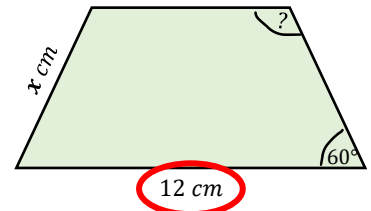
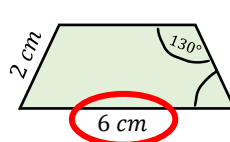


Multiply the distance from the centre of corresponding vertices by the scale factor along the ray

Calculations in similar shapes

Don't forget that properties of shapes don't change with enlargements or in similar shapes

The two trapezium are similar find the missing side and angle



Corresponding sides identify the scale factor

$$\frac{12}{6} = 2$$

Scale Factor = 2

Calculate the missing side

Length (corresponding side) $\times$ scale factor

$$2\text{ cm} \times 2$$

$$x = 4\text{ cm}$$

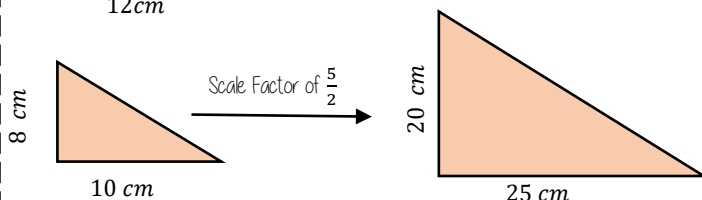
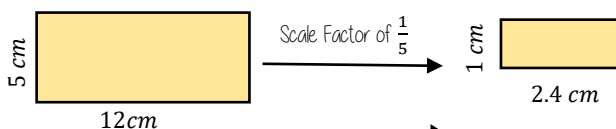
Enlargement does not change angle size

Calculate the missing angle

Corresponding angles remain the same 130°

Positive fractional scale factor

With a scale factor between 0 and 1 it makes the shape smaller



Year 11 Foundation Term 1

Vectors

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand and represent vectors
- Use and read vector notation
- Draw and understand vectors multiplied by a scalar
- Draw and understand addition of vectors
- Draw and understand addition and subtraction of vectors

Keywords

Direction: the line our course something is going

Magnitude: the magnitude of a vector is its length

Scalar: a single number used to represent the multiplier when working with vectors

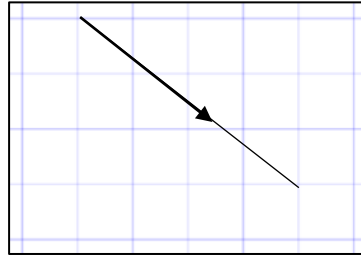
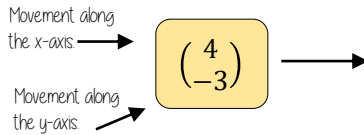
Column vector: a matrix of one column describing the movement from a point

Resultant: the vector that is the sum of two or more other vectors

Parallel: straight lines that never meet

Understand and represent vectors

Column vectors have been seen in translations to describe the movement of one image onto another



Vectors show both direction and magnitude

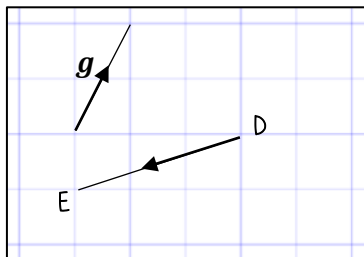
The arrow is pointing in the direction from starting point to end point of the vector.

The direction is important to correctly write the vector

The magnitude is the length of the vector (This is calculated using Pythagoras theorem and forming a right-angled triangle with auxiliary lines)

The magnitude stays the same even if the direction changes

Understand and represent vectors



Vector notation $\overrightarrow{DE}$ is another way to represent the vector joining the point D to the point E

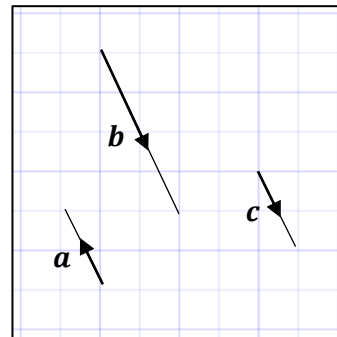
$$\overrightarrow{DE} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$$

The arrow also indicates the direction from point D to point E

Vectors can also be written in bold lower case so $\mathbf{g}$ represents the vector $\mathbf{g} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Vectors multiplied by a scalar

Parallel vectors are scalar multiples of each other



$$\mathbf{b} = 2 \times \mathbf{c} = 2\mathbf{c}$$

Multiply $\mathbf{c}$ by 2 this becomes $\mathbf{b}$.
The two lines are parallel

$$\mathbf{a} = -1 \times \mathbf{c} = -\mathbf{c}$$

The vectors $\mathbf{a}$ and $\mathbf{c}$ are also parallel. A negative scalar causes the vector to reverse direction

$$\mathbf{b} = -2 \times \mathbf{a} = -2\mathbf{a}$$

$$\mathbf{a} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

Addition of vectors

$$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\overrightarrow{BC} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

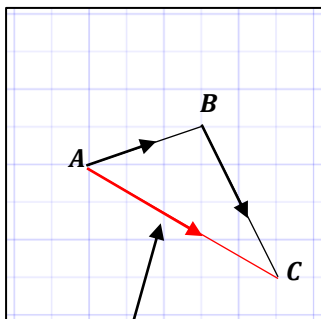
$$\overrightarrow{AB} + \overrightarrow{BC}$$

$$= \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} 3+2 \\ 1+(-4) \end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

Look how this addition compares to the vector $\overrightarrow{AC}$



The resultant

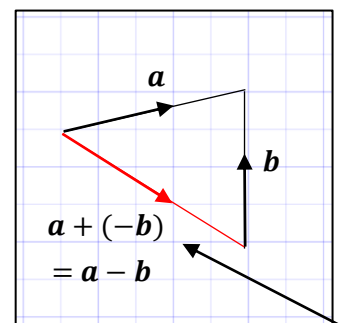
$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

Addition and subtraction of vectors

$$\mathbf{a} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$$\mathbf{b} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

$$\mathbf{a} + (-\mathbf{b}) = \begin{pmatrix} 5+(-0) \\ 1+(-4) \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$$



$$\mathbf{a} + (-\mathbf{b}) = \mathbf{a} - \mathbf{b}$$

The resultant is $\mathbf{a} - \mathbf{b}$ because the vector is in the opposite direction to $\mathbf{b}$ which needs a scalar of -1