

Year 10 Hybrid Term 1

Equations and Inequalities

What do I need to be able to do?

By the end of this unit you should be able to:

- Form and solve equations and inequalities
- Represent and interpret solutions on a number line as inequalities
- Draw straight line graphs and find solutions to equations
- Form and solve equations and inequalities with unknowns on both sides

Keywords

Solution: a value we can put in place of a variable that makes the equation true

Variable: a symbol for a number we don't know yet

Equation: an equation says that two things are equal – it will have an equals sign =

Expression: numbers, symbols and operators grouped together to show the value of something

Identity: An equation where both sides have variables that cause the same answer includes $\equiv$

Linear: an equation or function that is the equation of a straight line

Intersection: the point that two lines meet

Inequality: an inequality compares two values showing if one is greater than, less than or equal to another.

Solve equations R

$$2x + 4 = 30$$

$$6x + 12 = 30$$

$$6x = 18$$

$$x = 3$$

$$3(2x + 4) = 30$$

Expand the brackets

$$6x + 12 = 30$$

-12

-12

$$6x = 18$$

-6

-6

$$x = 3$$

Substitute to check your answer.
This could be negative or a fraction or decimal

Form and solve inequalities R



Two more than treble my number is greater than 11

Form

$$x \rightarrow x3 \rightarrow +2 \rightarrow 11$$

$$3x + 2 > 11$$

Solve

$$x \leftarrow -3 \leftarrow -2 \leftarrow 11$$

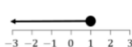
$$x > 3$$

Solutions on a number line



$$x < 1$$

Both represent values less than 1



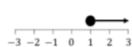
$$x \leq 1$$

Includes the value 1



$$x > 1$$

Both represent values more than 1



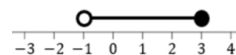
$$x \geq 1$$

Includes the value 1

● Includes the value it sits above

○ Does NOT include the value it sits above

Values less than or equal to 3 but also more than -1



$$-1 < x \leq 3$$

This includes the integer values 0, 1, 2, 3

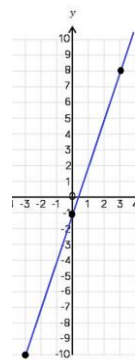
Plotting straight line graphs R

$$y = 3x - 1$$

x	-3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

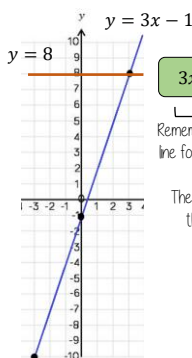
Find solutions graphically

For linear equations there is only one point the graph meets the x value

$$x = 2$$

$$y = 4$$

These two lines will cross at (2,4) because they are just x and y – they are parallel to axes and meet in one place



$$3x - 1 = 8$$

Remember equation of a line format is $y = mx + c$

The solution is the point the two lines meet **(3, 8)**

Equations: unknown on both sides R

$$8x + 5 = 4x + 13$$

$$8x + 5 = 4x + 13$$

$$4x + 5 = 13$$

$$4x = 8$$

$$8x + 5 = 4x + 13$$

$$-4x \quad -4x$$

$$4x + 5 = 13$$

$$-5 \quad -5$$

$$4x = 8$$

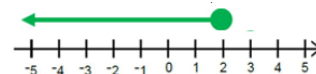
$$\div 4 \quad \div 4$$

$$x = 2$$

Inequalities: unknown on both sides

$$8x + 5 \leq 4x + 13$$

$$x \leq 2$$



Only value 2 or less will satisfy this inequality

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Probability

What do I need to be able to do?

By the end of this unit you should be able to:

- Add, Subtract and multiply fractions
- Find probabilities using likely outcomes
- Use probability that sums to 1
- Estimate probabilities
- Use Venn diagrams and frequency trees
- Use sample space diagrams
- Calculate probability for independent events
- Use tree diagrams

Keywords

Event: one or more outcomes from an experiment

Outcome: the result of an experiment

Intersection: elements (parts) that are common to both sets

Union: the combination of elements in two sets

Expected Value: the value/ outcome that a prediction would suggest you will get

Universal Set: the set that has all the elements

Systematic: ordering values or outcomes with a strategy and sequence

Product: the answer when two or more values are multiplied together.

Add, Subtract and multiply fractions

Addition and Subtraction

$$\frac{4}{5} - \frac{2}{3}$$

$$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$$

Use equivalent fractions to find a common multiple for both denominators

Multiplication

$$\frac{3}{4} \times \frac{2}{3}$$

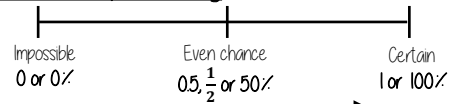
$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$$

Modelled: 3 (rows), 4 (columns)

Parts shaded

Total number of parts in the diagram

Likelihood of a probability



The more likely an event the further up the probability it will be in comparison to another event (it will have a probability closer to 1)

Sum to 1

Probability is always a value between 0 and 1

The probability of getting a blue ball is $\frac{1}{5}$
 $\therefore$ The probability of NOT getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is 1

Experimental data

Theoretical probability

What we expect to happen

Experimental probability

What actually happens when we try it out

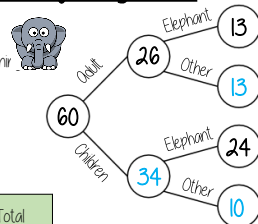
The more trials that are completed the closer experimental probability and theoretical probability become

The probability becomes more accurate with more trials.
 Theoretical probability is proportional

Tables, Venn diagrams, Frequency trees

Frequency trees

60 people visited the zoo one Saturday morning. 26 of them were adults. 13 of the adults' favourite animal was an elephant. 24 of the children's favourite animal was an elephant.



Frequency trees and two-way tables can show the same information

The total columns on two-way tables show the possible denominators

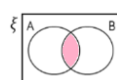
$$P(\text{adult}) = \frac{26}{60}$$

$$P(\text{Child with favourite animal as elephant}) = \frac{13}{37}$$

Two-way table

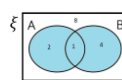
	Adult	Child	Total
Elephant	13	24	37
Other	13	10	23
Total	26	34	60

Venn diagram



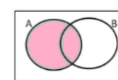
in set A AND set B

$$P(A \cap B)$$



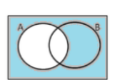
in set A OR set B

$$P(A \cup B)$$



in set A

$$P(A)$$



NOT in set A

$$P(A')$$

Sample space

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1H	2H	3H	4H	5H	6H
T	1T	2T	3T	4T	5T	6T

$$P(\text{Even number and tails}) = \frac{3}{12}$$

Independent events

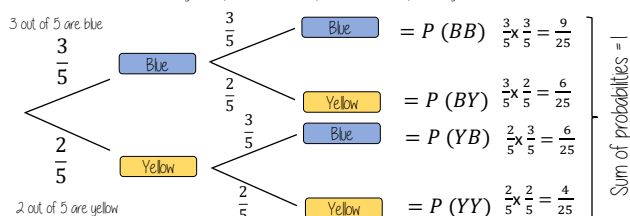
The outcome of two events happening. The outcome of the first event has no bearing on the outcome of the other

$$P(A \text{ and } B) = P(A) \times P(B)$$

Tree diagram for independent event

Isobel has a bag with 3 blue counters and 2 yellow. She picks a counter and replaces it before the second pick.

Because they are replaced the second pick has the same probability

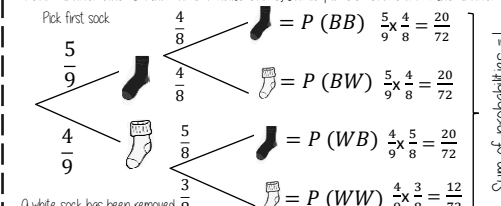


Dependent events

Tree diagram for dependent event

The outcome of the first event has an impact on the second event

A sock drawer has 5 black and 4 white socks. Jamie picks 2 socks from the drawer.



NOTE: as "socks" are removed from the drawer the number of items in that drawer is also reduced $\therefore$ the denominator is also reduced for the second pick.

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Percentages

What do I need to be able to do?

By the end of this unit you should be able to:

- Convert and compare FDP
- Work out percentages of amounts
- Increase/ decrease by a given percentage
- Express one number as a percentage
- Calculate simple and compound interest
- Calculate repeated percentage change
- Find the original value
- Solve problems with growth and decay

Keywords

Exponent: how many times we use a number in multiplication. It is written as a power.

Compound interest: calculating interest on both the amount plus previous interest.

Depreciation: a decrease in the value of something over time.

Growth: where a value increases in proportion to its current value such as doubling.

Decay: the process of reducing an amount by a consistent percentage rate over time.

Multiplier: the number you are multiplying by.

Equivalent: of equal value.

Compare FDP

Comparisons are easier in the same format.

70/100 → This also means 70 ÷ 100 → 70 out of 100 squares → 70 'hundredths' = 7 'tenths' 0.7 → 70 hundredths = 70%.

Using a calculator: $\frac{70}{100} = 0.7$ → Convert to a decimal → $\times 100$ converts to a percentage.

Be careful of recurring decimals:
e.g. $\frac{1}{3} = 0.333333$
 $\frac{1}{3} = 0.\dot{3}$ (The dot above the 3)

Fraction/ Percentage of amount

Find $\frac{3}{5}$ of £60

Remember: $\frac{3}{5} = 60\% = 0.6$

10% of £60 = £6
50% of £60 = £30
60% of £60 = £36

Remember: $\frac{3}{5} = 60\% = 0.6$
60% of £60 = $0.6 \times 60 = £36$

Percentage increase/decrease

100% → 42% → Decrease by 58% → 100% - 58% = 42%
Multiplier: Less than 1
 $100 - 0.58 = 0.42$

100% → 12% → Increase by 12% → 100% + 12% = 112%
Multiplier: More than 1
 $100 + 0.12 = 1.12$

Express as a percentage

27 per every 50 shaded → $\frac{27}{50} \times 100 = 54\%$
54 per every 100 shaded → $\frac{54}{100} \times 100 = 54\%$

$\frac{13}{30} \times 100 = 43.333\ldots\%$
→ 43%

Can't use equivalence easily to find 'per hundred' → Decimal percentages are still a percentage.

Simple and compound interest

Simple Interest: James invests £2000 at 5% simple interest. $\frac{100\%}{5\%} = 20$ → £100 → The original value increases by this amount every year.

Compound Interest: Tess invests £100 at 10% compound interest for 3 years. The multiplier 1.10 repeats each year.

£100 → 10% → Y1 £110 → 10% → Y2 £121 → 10% → Y3 £132.10

Repeated percentage change

Compound Interest: Tess invests £100 at 10% compound interest for 3 years. $£100 \times 1.10 \times 1.10 \times 1.10 = £132.10$

Depreciation: Depreciation calculations use multipliers less than 1. Multipliers are commutative — an overall multiplier effect can be calculated by combining the multipliers separately.
e.g. Increase of 10% then a reduction of 10% → $1.10 \times 0.90 = 0.99$ → The multiplier.

Growth and decay

Compound growth: The values increase exponentially. The constant multiplier is more than one.

Compound decay: The values get closer to 0. The constant multiplier is less than one.

Compound growth and compound decay are exponential graphs.

Find the original value

Percentage calculations: Original amount $\times$ Multiplier = Final Value.

In a test Lucy scored 60% of her questions correctly. Her score was 24. How many questions were on the test?
Original $\times 0.6 = 24$ → $24 \div 0.6 = 40$ marks. Total questions on test = 40.

A car sold for a profit £3000 with a profit of 20%. How much was the car originally?
Original $\times 1.2 = 3000$ → $3000 \div 1.2 = £2500$.

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Compound Measures

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve speed, distance, time questions
- Use distance time graphs
- Solve density, mass, volume problems
- Solve flow problems
- Use flow graphs
- Interpret rates of change and their units

Keywords

Convert: change

Mass: a measure of how much matter is in an object. Commonly measured by weight

Origin: the coordinate (0, 0)

Volume: the amount of 3D space a shape takes up

Substitute: putting numbers where letters are – replacing numbers into a formula

Speed, Distance, Time

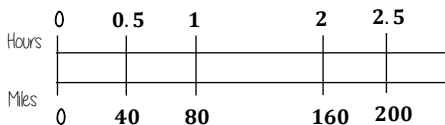
'per' for every

e.g. 80 miles per hour (mph)

Travel 80 miles every hour

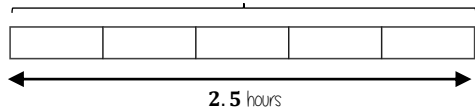
$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

You can use a double number line to help you calculate distance



e.g. A boat travels at a constant speed for 2.5 hours. It travels 300 miles.

300 miles



Bar models can help to calculate mph

Each part is half an hour
Each part is 60 miles

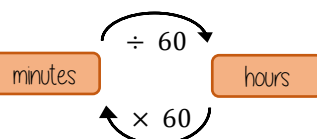


Speed, Distance, Time

Before calculations – make sure you are working in the same units as the speed

Learn or learn how to rearrange the formula for speed, distance and time

Substitute in the variables given



$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\text{distance} = \text{speed} \times \text{time}$$

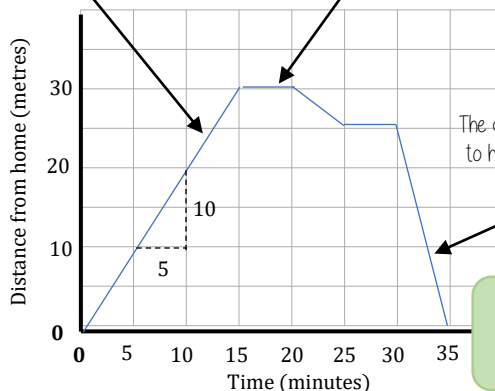
Distance – Time graphs

The steeper a gradient the faster the speed

$$\frac{10}{5} = 2 \text{ metres per min}$$

Gradient = speed

Horizontal lines represent staying still



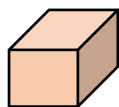
Units are important
Metres per minute

Density, Mass, Volume

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{volume} = \frac{\text{mass}}{\text{density}}$$

$$\text{mass} = \text{volume} \times \text{density}$$



$$\text{volume of prism} = \text{Area of cross section} \times \text{Depth}$$

Flow problems & graphs



This will fill at a constant rate, then as the space decreases it will speed up and the neck of the bottle fill at a faster constant speed



The cylinder will fill at a constant speed



Units are important
Ensure any volume calculations are the same unit as the rate of flow

Rates of change & units

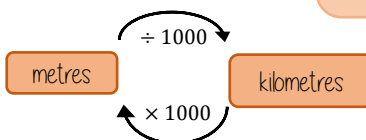
Common rates of change relationships

Revisit your conversions between units of length and capacity

Speed: miles per hour

Exchange rates: euros per pounds

Density: mass per volume



Year 10 Hybrid Term 1

Pythagoras' Theorem

What do I need to be able to do?

By the end of this unit you should be able to:

- Use square and cube roots
- Identify the hypotenuse
- Calculate the hypotenuse
- Find a missing side in a Right angled triangle
- Use Pythagoras' theorem on axes
- Explore proofs of Pythagoras' theorem

Keywords

Square number: the output of a number multiplied by itself

Square root: a value that can be multiplied by itself to give a square number

Hypotenuse: the largest side on a right angled triangle. Always opposite the right angle.

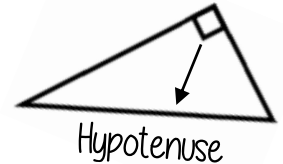
Opposite: the side opposite the angle of interest

Adjacent: the side next to the angle of interest

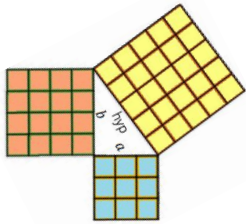
Squares and square roots

				This can also be written as 6^2						$\sqrt{\quad}$ is the square root symbol
1 x 1	2 x 2	3 x 3	4 x 4	5 x 5	6 x 6	7 x 7	8 x 8	9 x 9	10 x 10	eg $\sqrt{64} = 8$ Because $8 \times 8 = 64$
1	4	9	16	25	36	49	64	81	100	
Square numbers										

Identify the hypotenuse



Determine if a triangle is right-angled



If a triangle is right-angled, the sum of the squares of the shorter sides will equal the square of the hypotenuse.

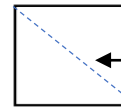
$$a^2 + b^2 = \text{hypotenuse}^2$$

eg $a^2 + b^2 = \text{hypotenuse}^2$

$$\begin{aligned} 3^2 + 4^2 &= 5^2 \\ 9 + 16 &= 25 \end{aligned}$$

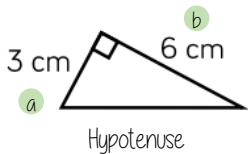
Substituting the numbers into the theorem shows that this is a right-angled triangle

The hypotenuse is always the longest side on a triangle because it is opposite the biggest angle.



Polygons can still have a hypotenuse if it is split up into triangles and opposite a right angle

Calculate the hypotenuse



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

1 Substitute in the values for a and b

$$3^2 + 6^2 = \text{hypotenuse}^2$$

$$9 + 36 = \text{hypotenuse}^2$$

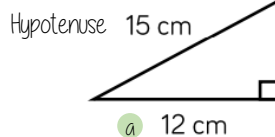
$$45 = \text{hypotenuse}^2$$

$$\sqrt{45} = \text{hypotenuse}$$

$$6.71\text{cm} = \text{hypotenuse}$$

2 To find the hypotenuse square root the sum of the squares of the shorter sides

Calculate missing sides



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

$$12^2 + b^2 = 15^2$$

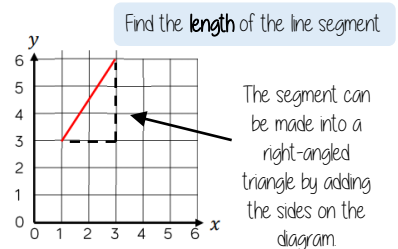
1 Substitute in the values you are given

$$144 + b^2 = 225$$

Rearrange the equation by subtracting the shorter square from the hypotenuse squared

$$\begin{aligned} \text{Square root to find the length of the side} \quad & \left\{ \begin{aligned} b^2 &= 111 \\ b &= \sqrt{111} = 10.54 \text{ cm} \end{aligned} \right. \end{aligned}$$

Pythagoras' theorem on a coordinate axis



The line segment is the hypotenuse

$$a^2 + b^2 = \text{hypotenuse}^2$$

The lengths of a and b are the sides of the triangle.

Be careful to check the scale on the axes

Year 10 Hybrid Term 1

Trigonometry

What do I need to be able to do?

By the end of this unit you should be able to:

- Work fluently with hypotenuse, opposite and adjacent sides
- Use the tan, sine and cosine ratio to find missing side lengths
- Use the tan, sine and cosine ratio to find missing angles
- Calculate sides using Pythagoras' Theorem

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)

Scale Factor: the multiplier of enlargement

Constant: a value that remains the same

Cosine ratio: the ratio of the length of the adjacent side to that of the hypotenuse. The sine of the complement

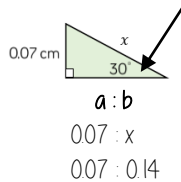
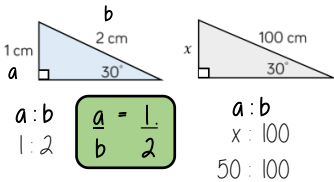
Sine ratio: the ratio of the length of the opposite side to that of the hypotenuse.

Tangent ratio: the ratio of the length of the opposite side to that of the adjacent side.

Inverse: function that has the opposite effect.

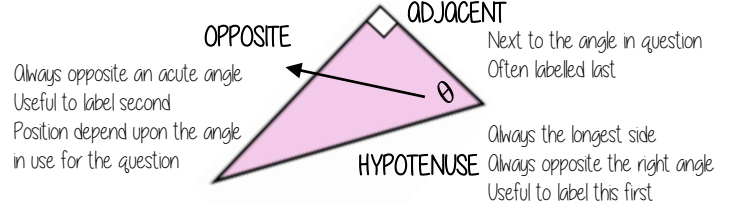
Hypotenuse: longest side of a right-angled triangle. It is the side opposite the right-angle.

Ratio in right-angled triangles



Hypotenuse, adjacent and opposite

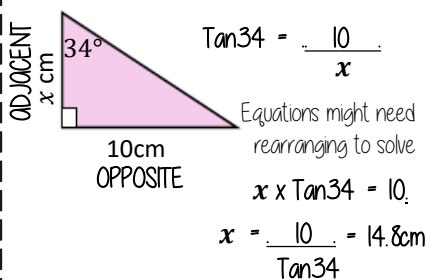
ONLY right-angled triangles are labelled in this way



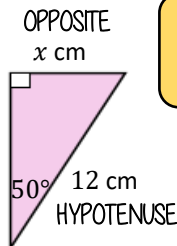
Tangent ratio: side lengths

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Substitute the values into the tangent formula



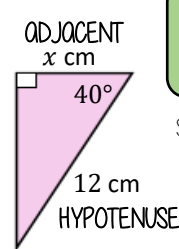
Sin and Cos ratio: side lengths



$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$$

NOTE

The $\sin(x)$ ratio is the same as the $\cos(90-x)$ ratio

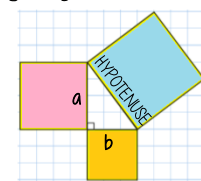


$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$$

Substitute the values into the ratio formula

Equations might need rearranging to solve

Pythagoras theorem



$$\text{Hypotenuse}^2 = a^2 + b^2$$

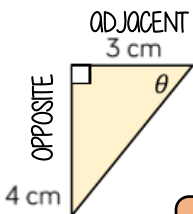
This is commutative — the square of the hypotenuse is equal to the sum of the squares of the two shorter sides

Places to look out for Pythagoras

- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Any length made from a right angles

Sin, Cos, Tan: Angles

Inverse trigonometric functions



Label your triangle and choose your trigonometric ratio

Substitute values into the ratio formula

$$\theta = \tan^{-1} \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\theta = \sin^{-1} \frac{\text{opposite side}}{\text{hypotenuse side}}$$

$$\theta = \cos^{-1} \frac{\text{adjacent side}}{\text{hypotenuse side}}$$

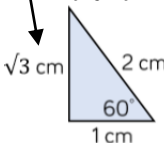
$$\tan \theta = \frac{4}{3}$$

$$\theta = \tan^{-1} \frac{4}{3}$$

$$\theta = 36.9^\circ$$

Key angles

This side could be calculated using Pythagoras



$$\tan 30 = \frac{1}{\sqrt{3}}$$

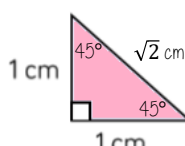
$$\tan 60 = \sqrt{3}$$

$$\cos 30 = \frac{\sqrt{3}}{2}$$

$$\cos 60 = \frac{1}{2}$$

$$\sin 30 = \frac{1}{2}$$

$$\sin 60 = \frac{\sqrt{3}}{2}$$



$$\tan 45 = 1$$

$$\cos 45 = \frac{1}{\sqrt{2}}$$

$$\sin 45 = \frac{1}{\sqrt{2}}$$

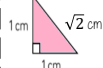
Because trig ratios remain the same for similar shapes you can generalise from the following statements

Key angles 0° and 90°

$$\tan 0 = 0$$

$$\tan 90$$

This value cannot be defined — it is impossible as you cannot have two 90° angles in a triangle



$$\sin 0 = 0$$

$$\sin 90 = 1$$

$$\cos 0 = 1$$

$$\cos 90 = 0$$